A Lesson Plan Intervention for Preservice Elementary Teachers: Bridging Mathematical Content from a Methods Course to Student Teaching

A Dissertation
SUBMITTED TO THE FACULTY OF THE
UNIVERSITY OF MINNESOTA
BY

Timothy James Morrison

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Dr. Erin Baldinger, Advisor
Dr. Kathleen Cramer, Advisor

AUGUST 2019

Acknowledgements

I would like to express my sincerest thanks to my advisors, Dr. Kathleen Cramer and Dr. Erin Baldinger. Dr. Cramer was with me from the beginning of my doctoral journey and worked with me even after her retirement. I sincerely appreciated her "bigger picture" knowledge of mathematics research and how she was able to steer my own interests to add to the literature. Thank you for hanging in there with me!

Dr. Baldinger joined the STEM Department at the University of Minnesota and soon became my co-advisor. Dr. Baldinger helped me see my potential as a researcher and gave me numerous insights on how research can be done, especially qualitative research. She helped me see how research is connected and how these connections add to the richness of mathematics and mathematics instruction. Dr. Baldinger was of great encouragement, especially during some difficult times in the dissertation writing process. Your patience and wisdom kept me going, and for this I am deeply grateful.

I would like to thank my committee chair, Dr. David O'Brien. David's course in qualitative research methods was one of my favorites. Dr. O'Brien demonstrated expertise in the areas in which I needed help and was instrumental in my understanding of how qualitative research can be gathered and analyzed. I appreciate his dedication to ensuring that research participants' stories were accurately displayed and honored. Thank you for helping me bring out the stories of Jennie, Penny, and Eleanor!

To Dr. Bhaskar Upadhyay I also say "thank you" for your positivity in the research process. Your instruction was amazing and I appreciated your willingness to help me along the way. To Dr. Terry Wyberg I am also grateful. Your zeal for helping preservice teachers be better mathematics teachers is inspiring to anyone who has had a course with you. You have helped make me a better mathematics methods instructor.

I would also like to thank my fellow instructors, Dr. Dawn Locklear, Dr. Lynne Shenk, Dr. Aeisha Thomas, and Department Chair Scotti Moats, all of whom gave me advice and encouragement along the way. A special thanks goes to Dr. Don Bouchard, on whose broad shoulders I constantly leaned. Thank you, friend.

I am grateful to Dr. Jim Brickwedde for introducing me to the CGI framework when we were teaching in Hopkins, Minnesota.

Thanks also goes to my friends, Kelly Doyle, Lane Lackas, and Mike Walvatne. I needed your prayers and friendship in this adventure. Thanks to my brothers Mike, John, Dave, and Chris Morrison - let's go do some serious fishing!

Finally, I would like to acknowledge God's grace in helping me through this wonderful and difficult journey. When the trail got tough, Christ kept me encouraged and faithful to the task.

Dedication

This dissertation is dedicated to my amazing wife, Wendy Morrison, who was at my side through thick and thin. I appreciate how you did not give up on me in the journey and kept me focused. Thanks for understanding why I often zoned out in the middle of conversations and had to be asked questions over again! I love you tons!

I am so grateful to my daughter, Melissa Morrison, who has been there with me on this journey, often delivering Chipotle when she knew I needed a break. Your intelligence and creativity are only superseded by your genuine love for Mom and me. I love you and am so glad God put you in our family! With your amazing Yorkie, Toby, aka PhDog, I always had the best break times.

Thank you, Mom, for believing in me; and thank you, Dad, for demonstrating what integrity and sacrifice looks like.

Abstract

When preservice elementary teachers (PSETs) begin their student teaching placements, they often encounter challenges in bridging from their mathematics methods course content and pedagogies to their student teaching classrooms. In this study I examined the experiences of three PSETs at a Midwestern United States Christian college as they engaged in a lesson plan intervention (LPI) that integrated elements of Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Loef-Franke, Levi, & Empson, 2015) into their lesson plans and enacted lessons. I studied the LPI to examine its implementation as a bridge from mathematics methods courses to student teaching practices. The purpose of the LPI was to help PSETs elicit, interpret, and leverage student mathematical thinking during large-group, problem-solving lessons to build students' number sense. Using data from lesson observations, LPI training sessions, lesson plan analysis, interviews, and conversations, I used a constant comparative approach to understand PSETs' experiences (Cohen, Manion, & Morrison, 2011; Glaser & Strauss, 2017; Miles, Huberman, & Saldana, 2014). Findings indicate that the LPI had differing effects on PSETs' practices in integrating CGI elements into their lesson plans and enacted lessons in their student teaching placement classrooms.

Table of Contents

Acknowledgements.......... i
Dedication.......... ii
Abstract.......... iii
Table of Contents.......... iv
List of Tables.......... x
List of Figures.......... xi
Chapter One: Introduction.......... 1
Teaching for Mathematical Sense Making.......... 2
Sense Making Via Student Thinking.......... 5
Undergraduate Teacher Preparation and Student Thinking.......... 7
Connecting Methods Course Content with Student Teaching.......... 9
Leveraging Student Thinking Through CGI.......... 10
Research Questions to Explore Student Thinking.......... 13
A Lesson Plan Intervention to Explore Student Thinking.......... 14
Overview of the Study.......... 20
Sections of the LPI study.......... 20
Outline of Thesis.......... 21
Chapter Two: Review of Literature.......... 22
Bridging Mathematics Methods Coursework with Student Teaching.......... 22
Student Teaching.......... 25
Interventions with PSETs.......... 28
PSETs' Lesson Planning.......... 30
Mathematical Sense Making.......... 31
PSETs and Student Mathematical Thinking.......... 32
Cognitively Guided Instruction.......... 36
CGI in Practice.......... 40
Limitations to CGI.......... 42
Summary of the Research.......... 44
Chapter Three: Research Methods.......... 46
Research Design.......... 46
Research Questions.......... 47
Qualitative Methodology.......... 47
Components of the LPI Study.......... 50
Steps of Research.......... 51

Participants 52
Mathematics Methods Course 52
Participant Selection 53
School and Classroom Context 54
Summary Chart of Structure of Data Collection 56
Research questions and data collection 56
Two Pre-LPI Assessments, CGI Framework, and Self-Efficacy 59
CGI framework knowledge assessment 59
CGI knowledge assessment design 60
Self-efficacy assessment 61
Self-efficacy assessment design 61
Pre-LPI lesson (Lesson One) 62
The Lesson Plan Intervention 63
Framework for the LPI 63
LPI first session 64
LPI second session 66
LPI lesson plan two 68
LPI enacted lesson three 68
Researcher Background and Stance 68
Data Sources 70
Coding of CGI elements 70
Data Analysis 73
Trustworthiness 75
From Purpose to Enacted Research 79
Chapter Four: Findings 81
Chapter Four Overview 81
Research Questions One and Two 83
Jennie as a Case 84
Jennie's utilization of CGI elements. 86
Jennie's use of broadcasting 88
Broadcasting in lesson plan one and enacted lesson one 88
Broadcasting in LPI sessions 89
Broadcasting in lesson plan two and enacted lesson two 91
Broadcasting in lesson plan three and enacted lesson three 92
Changes in Jennie's broadcasting 94

Jennie's use of invites multiple strategies ... 94
Invites multiple strategies in lesson plan one and enacted lesson one ... 95
Invites multiple strategies in the LPI. ... 95
Invites multiple strategies in lesson plan two and enacted lesson two ... 97
Invites multiple strategies in lesson plan three and lesson three ... 99
Changes in Jennie's use of invites multiple strategies ... 100
Jennie and invites reflection ... 100
Invites reflection in LPI sessions ... 101
Invites reflection in lesson plan two and enacted lesson two ... 102
Invites reflection in lesson plan three and enacted lesson three ... 103
Changes in Jennie's use of invites reflection ... 104
Jennie's use of builds on student starting points. ... 105
Builds on student starting points in lesson plan one and lesson one ... 105
Builds on student starting points in the LPI sessions ... 106
Builds on student starting points in lesson plan two and enacted lesson two. ... 107
Builds on student starting points in lesson plan three and enacted lesson three ... 109
Changes in Jennie's use of builds on students' starting points ... 110
Jennie and Research Questions One and Two ... 111
Jennie and Research Question Three ... 112
Changes in Jennie's Teaching Practices ... 113
Penny as a Case ... 115
Penny's utilization of CGI elements. ... 117
Penny's use of broadcasting ... 119
Broadcasting in lesson plan one and enacted lesson one ... 119
Broadcasting in LPI sessions ... 120
Broadcasting in lesson plan two and enacted lesson two ... 121
Broadcasting in lesson plan three and enacted lesson three ... 122
Changes in Penny's use of broadcasting ... 123
Penny's use of invites multiple strategies ... 124
Invites multiple strategies in lesson plan one and enacted lesson one ... 124
Invites multiple strategies in LPI sessions ... 125
Invites multiple strategies in lesson plan two and enacted lesson two ... 126
Invites multiple strategies in lesson plan three and enacted lesson thre ... 126
Changes in Penny's use of invites multiple strategies ... 128
Penny's use of invites reflection ... 129

Invites reflection in lesson plan one and enacted lesson one 129
Invites reflection in LPI sessions 129
Invites reflection in lesson plan two and enacted lesson two 130
Invites reflection in lesson plan three and enacted lesson three 131
Changes in Penny's use of invites reflection.. 132
Penny's use of willing to struggle 133
Willing to struggle in lesson plan one and enacted lesson one...................................... 133
Willing to struggle in LPI sessions 134
Willing to struggle in lesson plan two and enacted lesson two...................................... 135
Willing to struggle in lesson plan three and enacted lesson three................................. 137
Changes in Penny's use of willing to struggle 138
Penny and research questions one and two.. 139
Penny and research question three.. 140
Changes in Penny's teaching practices 141
Eleanor as a Case 143
Eleanor's utilization of CGI framework elements. 146
Eleanor's use of broadcasting.. 148
Broadcasting in lesson plan one and enacted lesson one 148
Broadcasting in LPI sessio 149
Broadcasting in lesson plan two and enacted lesson two 150
Broadcasting in lesson plan three and enacted lesson three 151
Changes in Eleanor's broadcasting 152
Eleanor's use of invites multiple strategies 153
Invites multiple strategies in lesson plan one and enacted lesson one 153
Invites multiple strategies in LPI sessions... 154
Invites multiple strategies in lesson plan two and enacted lesson two 154
Invites multiple strategies in lesson plan three and enacted lesson three 155
Changes in Eleanor's invites multiple strategies 157
Eleanor's use of invites reflection... 157
Invites reflection in lesson plan one and enacted lesson one. 157
Invites reflection in LPI sessions 158
Invites reflection in lesson plan two and enacted lesson two 158
Invites reflection in lesson plan three and enacted lesson three................................... 159
Changes in Eleanor's invites reflection 159
Eleanor's use of pursues to completion 160

Pursues to completion in lesson plan one and enacted lesson one 160
Pursues to completion in LPI sessions. 161
Pursues to completion in lesson plan two and enacted lesson two 162
Pursues to completion in lesson plan three and enacted lesson three 164
Changes in Eleanor's pursues to completion 165
Eleanor and research questions one and two 166
Eleanor and research question three 167
Changes in Eleanor's teaching practices 169
Summary of Cases for Questions One and Two 171
Summary of Cases for Question Three 172
Chapter Five: Discussion 174
Summary of Procedures 176
Discussion of PSETs and Research Questions 177
Jennie and the LPI 177
Research question one 178
Research question two 183
Research question three 186
Summary of Jennie's experiences 187
Penny and the LPI 187
Research question one 188
Research question two 191
Research question three 195
Summary of Penny's experiences 196
Eleanor and the LPI 197
Research question one 197
Research question two 200
Research question three. 202
Summary of Eleanor's experiences 202
Conclusions 203
Importance of Study 206
Suggestions for Future Practice 208
Directions for Further Research 209
References 211
Appendices 235
Appendix A: Participant Self-Administered Pre-Assessment 236

Mathematics Self-Efficacy 236
Appendix B: Participant Self-Administered Pre-Assessment 237
Knowledge of CGI 237
Appendix C: College Boilerplate Lesson Plan 239
Appendix D: CGI Elements - Key Elements of Cognitively Guided Instruction 242
Appendix E: CGI Problem Types 244
Appendix F: CGI Solution Strategies 245
Appendix G: Interview Protocol 246
Appendix H: Omitted Research Questions 247

List of Tables

Table 1: Codes, designations, and descriptions from analyses of data, pp. 71-72

Table 2: Elements of CGI framework present in Jennie's lesson plans, p. 86

Table 3: Elements of CGI framework in Jennie's enacted lessons, p. 87

Table 4: Elements of CGI framework in Penny's lesson plans, p. 117

Table 5: Elements of CGI framework in Penny's enacted lessons, p. 118

Table 6: Elements of CGI framework in Eleanor's lesson plans, p. 146

Table 7: Elements of CGI framework in Eleanor's enacted lessons, p. 147

Table 8: Presence or non-presence of six CGI elements in PSETs' lesson plan interventions, lesson plans, and enacted lessons, p. 171

List of Figures

Figure 1: Overview of the five sections of the study, p. 20

Figure 2: Overview of the sections of the study, p. 50

Figure 3: Research question, data source, and description of data source collection for all six research questions, pp. 56-58

Figure 4: Example of student's solution strategy for solving 8 + 8 = ___, p. 65

Figure 5: Analytic notes framework, p. 70

Figure 6: Analysis and convergence of data for this intervention study, p. 74

Figure 7: Jennie's visual representation of two addends and a sum - what she described as a "math mountain", p. 98

Chapter One: Introduction

This study explored a lesson plan intervention (LPI) for preservice elementary teachers (PSETs) during their student teaching placements. The purpose of the LPI was to equip PSETs to create lessons plans that, when enacted, would build number sense in their first grade students. Building number sense in children is important and is a vital component to elementary students' academic success (National Council of Teachers of Mathematics, 2000). To equip PSETs to build sense making opportunities in their placement classrooms, the LPI utilized Cognitively Guided Instruction (CGI) as a framework for helping the PSETs elicit, interpret, and utilize first graders' mathematical thinking to build their number sense. CGI is a framework for understanding children's mathematical thinking in early number. Understanding student mathematical thinking is an essential factor in helping teachers make informed decisions in the real-time learning environments of classrooms (Carpenter, Fennema, & Franke, 1996; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996). As teachers gain knowledge of how children think mathematically, especially concepts that are related to problem solving, teachers become more aware of how students naturally move from direct modeling structures of quantities to more abstract strategies (Carpenter et al., 2015). Instruction that values these progressions is situated to better elicit and interpret student thinking to help students explore how the quantities in the problem interact with each other. "We found that learning to understand the development of children's mathematical thinking could lead to fundamental changes in teachers' beliefs and practices and that these changes were reflected in students' learning" (Carpenter et al., 2015, p. 200).

One way to effectively explore children's mathematical thinking is through the discourses that arise when children are tasked with solving problems. Problem solving is a central component to students' growth in understanding of quantities and how quantities relate to each other and can change over time. One avenue for solving mathematics problems is asking students to solve word problems. While engaged in word problems, mathematical concepts can be discussed in the natural setting that word problems initiate. As students' mathematical thinking during problem solving situations becomes known to teachers, teachers are better situated to help students explore their own, and others', solution strategies. "For students to learn mathematics with understanding, they must have opportunities to engage on a regular basis with tasks that focus on reasoning and problem solving and make possible multiple entry points and varied solution strategies" (National Council of Teachers of Mathematics, 2014, p. 34).

To explore PSETs' pursuit of student thinking, an LPI was implemented for each PSET using the mathematics framework of CGI. The LPI reviewed elements of CGI with the PSETs for the co-creation of an LPI lesson plan. CGI practices were a good fit for the study as they emphasized attending to and interpreting student thinking during problem solving. CGI framework was a central theoretical component of the PSETs' prior methods class and as such was the core feature of the intervention.

Teaching for Mathematical Sense Making

Student mathematical sense making is a priority in mathematics education (National Council of Teachers of Mathematics, 2014). Sense making takes place every day in a child's life when they try to solve a challenging problem, apply their understanding of a new idea, or explain their thinking to another student. Mathematical sense making occurs when students connect concepts for new purposes or for application with previously learned ideas. For example, sense making is evident when a student who correctly understands the concept of addition makes the connection that addition could be done repeatedly, and that multiplication seems to involve these repeated additions and could be used to solve a problem in a new, yet interrelated, way. As the student explores this repeated addition idea, mental connections are made that reinforce the relatedness of multiplication with repeated addition, and new knowledge is created in the child's mind. Sense making in this instance is the result of understanding the interrelatedness of addition with multiplication, thereby creating new knowledge. This creation of new knowledge is an important part of sense making.

Students engaged in sense making activities discover similarities and differences in conceptual structures and number relationships, resulting in an increasingly sophisticated understanding of how numbers relate to each other. Mathematically, sense making can be observed in students when "mathematical ideas 'feel' clear, logical, valid, or obvious" (Battista, 2017, p. 1). There is a sense that the child has grasped a new idea while searching for a solution to a problem or mathematical opportunity. Often, their learning is quite visible to teachers as well as to other students. With repeated exposure to learning opportunities, children will naturally engage in exploratory strategies to solve the problem. This is a kind of mathematical sense making.

Mathematical sense making begins at a very early age. For most children it is a natural process that often takes place while playing with toys, talking with peers, or singing a song. Though socio-culturalists and cognitive nativists disagree on some aspects of how children learn to count, most researchers agree sense making is an important stage in children's cognitive development (Stock, Desoete, & Roeyers, 2009). Children's counting ability varies from child to child, of course, but all children bring with them an intuitive sense of trying to understand quantities of the things they interact with on a daily basis (Carpenter & Fennema, 1992). These daily interactions provide a rich variety of things to count, subitize, sort, and organize. As children begin to solve mathematics problems, they approach tasks differently than adults, sometimes making surprising connections and sometimes missing what seem to be basic ideas and relationships. The skill of counting, for instance, is a basis for further mathematical learning (Carpenter & Fennema, 1992; Stock et al., 2009) and grows as students begin to see relationships among numbers. As children grow in their understanding of these relationships, multiple solution strategies emerge.

These new strategies are the products of children's growing number sense and show themselves through working with objects, drawings, real life situations, verbal symbols, and conversations (Lesh, Post, & Behr, 1987). Consider the following word problem: "Johnny had 14 gummy bears. He gave 6 of them away. How many does he have now?" A child can use a range of possible solution strategies. The child may start with 14 pieces and remove six of them, counting the remaining 8 pieces. The child could also start with 6 pieces and add more until she got to 14, knowing the missing addend is 8. As a third strategy, the child could compare a row of 6 pieces with a row of 14 and look for the difference between the two. Each solution strategy has a level of sophistication that indicates much about how the child thinks and understands numerical relationships. As children gain more experience solving problems, they become more sophisticated, adding to their knowledge base and problem solving strategies. This is a part of what mathematical sense making looks like in young children.

Sense making differs from rote learning in that sense making requires students to wrestle with conceptual knowledge (Darling-Hammond, 2010), resulting in mathematical fluency, whereas rote learning relies on procedures as foundations for learning. Cohen et al. (2011) comments that instruction commonly “is marked by little close attention to learners’ thinking and little effort to design instruction to advance it” (p. 27). While procedures (algorithms) are certainly an important slice of the mathematics pie, procedural fluency should not be confused with mathematics fluency, where many learning factors combine for sense making. While procedural fluency is a part of mathematical fluency, mathematical fluency is the descriptor for the larger picture of mathematical competency. Like a house of cards on a windy day, rote/procedural learning alone will fail a student in the challenge of an unfamiliar learning opportunity. Mathematical sense making values procedural fluency, but as a tool that comes from students’ conceptual understanding, and not vice-versa (National Council of Teachers of Mathematics, 2014).

Sense Making Via Student Thinking

To foster student sense making in mathematics, teachers use many strategies, including leveraging of student thinking (National Council of Teachers of Mathematics, 2014). In light of this, it is a priority in mathematics education to pursue student thinking so that teachers are better positioned to make sound instructional decisions (Levin, Hammer, & Coffey, 2009). To build sense making in children, an effective teacher “uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (National Council of Teachers of Mathematics, 2014, p. 10). Knowing how students solve problems helps teachers anticipate potential solution strategies and plan for meaningful discussions around those strategies (Carpenter et al., 2015; Carpenter & Fennema, 1992; Carpenter, Franke, Johnson, Turrou, & Wager, 2017). Students need to have "experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (National Council of Teachers of Mathematics, 2014, p. 5). Understanding student mathematical thinking is an important step for a teacher, setting the stage to use "mathematically significant pedagogical opportunities to build on student thinking" (Leatham, Peterson, Stockero, & Van Zoest, 2015, p. 308). Developing skill in leveraging student thinking to enhance instruction is an important goal for preparing future elementary teachers to become effective mathematics teachers.

PSETs' knowledge of student thinking helps them design lessons specifically for the building of sense making in students. Knowing how students solve problems allows a teacher to anticipate solution strategies and plan for meaningful discussions about the strategies (Carpenter et al., 2015). Stockero, Rupnow, and Pascoe (2015) discovered that an intervention aimed at helping preservice teachers notice student mathematical thinking through an analytical framework was successful. This framework for preservice teacher noticing is important in my study, as it also incorporates a framework for a similar task. As higher potential ideas are recognized by PSETs, they become starting points in productive mathematical discussions.

As student thinking is pursued and interpreted (Jacobs, Lamb, & Phillip, 2010), teachers must make decisions in real time, based on experiences with children, knowledge of mathematics content, and pedagogical skill. As students wrestle with mathematics problems, teachers have opportunities to practice mathematics talk with students to reveal their thinking to themselves as well as to other students (Chapin, O'Connor, & Anderson, 2013). Nothing short of a high wire act, this kind of instruction takes PSETs time to learn and does not come without a lot of time with students and instructor feedback. The start of this kind of student-centered pedagogy begins in methods courses where PSETs learn the concepts of exploring student thinking and start to practice it in their practicum placement classrooms.

Undergraduate Teacher Preparation and Student Thinking

Methods courses are designed to help PSETs make the difficult, but necessary, transition from guided teaching to independent teaching of students in placement classrooms (Stengel & Tom, 1996). Methods courses help preservice teachers learn the content and pedagogy necessary to teach effectively in the classroom and offer opportunities to build sense making capacities in their students.

However, there are disconnects that occur between ideas taught in methods courses and what is taught and observed in the field with regard to leveraging student thinking for sense making. Haefner and Zembal-Saul (2004) state that some preservice teachers do not see models of inquiry-based learning in their placement classrooms or methods courses. These models often are a companion to sense making activities. Raizen and Michelsohn (1994) reported that some pedagogies learned in elementary science methods courses do not readily transfer to classroom experiences because of differences in strategies between methods classes and what students see happening in their field experiences. For certain, there are real and common circumstances that hinder PSETs' successes in carrying over methods course material into their field placements: dissonance between university training and field placement expectations (Darling-Hammond, 2014; Edwards & Protheroe, 2003; Rose & Rogers, 2012; Valencia, Martin, Place, & Grossman, 2009). The good news is that methods courses often are successful in training PSETs to be effective, responsive, and competent new teachers (Abell & Bryan, 1997; Carrier, 2011; Huinker & Madison, 1997).

Effective, research-based instruction comes from methods courses (van Ingen, Alvarez McHatton, & Vomvoridi-Ivanovic, 2016) Some research suggests that PSETs are influenced by their methods courses in various ways (Smith & Gess Newsome, 2004). Micro-teaching is a common practice in methods classes and has been shown to help preservice teachers value feedback on their teaching (Zeichner, 2007). Completing methods courses increases preservice teachers' sense of efficacy (Stripling, Ricketts, Roberts, & Harlin, 2008), which touches on the self-efficacy pre-assessment given to the PSETs before this study started. PSETs' mathematical beliefs have been shown to change in elementary mathematics methods courses (Bahr, 2008). This is significant as beliefs are historically resistant to change. Similarly, methods course instructors have been shown to positively change preservice teachers' beliefs in teaching and technology integration (Molebash, 2004). Methods courses, however, need to be connected to student teachers' opportunities to practice what they have learned as they teach in their placement classrooms (Fehn & Koeppen, 1998). Bridging methods course content into student teaching is an important task in teacher education and is the primary problem for this research.

Connecting Methods Course Content with Student Teaching

For most undergraduate teacher preparation programs, student teaching is the capstone element of preservice teacher education. Student teaching placements "can develop a pre-service teacher's ability to assess a situation, make judgments, create goals, choose a course of action and reflect on its success" (Hertzog & O'Rode, 2011, p. 91). Student teaching offers a place where PSETs can safely implement theory they have learned in their methods courses, as student teachers encounter the many hindrances that affect how they teach during student teaching (Fehn & Koeppen, 1998; Valencia et al., 2009). Student teaching is a pivotal factor in preparing beginning teachers (Carter & Richardson-Koehler, 1989; Wilson, Floden, & Ferrini-Mundy, 2002) and is a key component of teacher training. It is here where PSETs make important decisions about how they will teach and what they specifically will enact in the future (Stengel & Tom, 1996). It is an important junction; student teaching is considered to be one of the most difficult experiences for PSETs to navigate in teacher education programs (Valencia et al., 2009).

When PSETs finally enter their student teaching classrooms, they are faced with challenging teaching issues (Fehn & Koeppen, 1998). Some of these challenges include: (a) teaching in another educator's classroom, (b) curricula nuances, (c) teaching program expectations, (d) supervisor and cooperating teacher expectations and assumptions, (e) PSETs' and cooperating teachers' knowledge of content and pedagogy, (f) K-12 students' resistance to new modes of teaching, (g) sparse feedback from supervisors, (h) classroom environment and discipline dynamics, and (i) few links to methods courses (Fehn & Koeppen, 1998). The last of these challenges, few links to methods courses, is the backdrop for this study. As PSETs engage in methods courses during their second, third, and fourth years of study, it can be a lengthy time lapse between methods courses and student teaching, making transfer of concepts and pedagogies difficult.

With most four year undergraduate teacher preparation programs, the majority of methods courses do not run concurrently with student teaching (Darling-Hammond, Hammerness, Grossman, Rust, & Shulman, 2005). This time gap is one of many challenging and complex issues facing student teachers (Valencia et al., 2009). To address the time gap between methods courses and student teaching, my research applied an intervention designed to re-integrate mathematics methods course material about student thinking into PSETs' student teaching placements. For this study, the researcher utilized CGI research, which is a body of knowledge encompassing student thinking in mathematical problem solving (Carpenter et al., 1996).

Leveraging Student Thinking Through CGI

CGI framework is a research-based model that connects student mathematical thinking with teachers' knowledge of content, pedagogy, and curriculum (Carpenter et al., 1996). CGI practices allow teachers to "interpret, transform, and reframe their informal or spontaneous knowledge about students' mathematical thinking" (Carpenter et al., 1996, p. 2). CGI's main tenet is that students have a natural, informal knowledge of mathematics that can be built upon for learning about early number concepts. The emphasis on student thinking, combined with specific information about different kinds of word problems and their solution strategies, makes CGI framework a highly effective and practical model for teaching. As a teacher gathers the thoughts of a student engaged in problem solving, the teacher leverages this thinking to ask pertinent questions about the concept being explored. Through intentional listening to students' responses, a dialogue is built that gives insight into the concepts, misconceptions, and numerical relationships the child is considering.

CGI is a framework that emphasizes guiding students to explore their intuitive thinking about how they might solve a problem and using that information to make *in the moment* instructional decisions (Carpenter et al., 2015; Jacobs et al., 2010; Van Zoest, Stockero, Leatham, Peterson, Atanga, & Ochieng, 2017). CGI is not a curriculum or scripted course of conversation with students. The following is a short example of how a CGI practitioner/teacher might approach our previous problem of "Sasha had 14 gummy bears. She gave 6 of them away. How many does she have now?" Because CGI practices value student thinking, the teacher could ask an open-ended question, such as, "Latisha, can you show me how you figured out this problem?" Latisha may start her explanation as she gathers 14 gummy bears saying, "Sasha has these 14 bears. I moved 6 of them. Now I have 8 bears." CGI practices would encourage the teacher to ask something like, "Why did you start with 14 bears?" Latisha could respond in several ways, but says, "Sasha had 14 bears and she sometimes gives some away. So I took the 6 she gave away and then I had to figure out how many she had left." Barring no other child-initiated thoughts, the teacher might ask, "Can you show me how you figured out how many she had left?" or "Why did you move 6 bears away from the group?" As the dialogue goes on, the teacher is constantly listening to the child and merging Latisha's thinking with solution strategies that children commonly use with this problem type.

With CGI practices, the teacher unpacks the child's thinking, being careful not to make assumptions or shift into direct instructional mode. CGI provides a framework for problem types and solution strategies (more on this in Chapter Three), helping the teacher further explore a child's thinking. In CGI framework terms, the Sasha word problem is called a *separate - result unknown* problem type. This type of problem has several typical solution strategies which children use, each with different degrees of mathematical sophistication. In this example, Latisha first gathered 14 bears and then moved 6 away. This is a less sophisticated solution strategy than others that are commonly used, so the teacher would purse the child's thinking accordingly. This kind of guiding does not tell Latisha how to solve the problem and so results in a more thorough exploration of what Latisha understands about the key idea at work in this scenario - that Latisha can keep track of the 14 bears as she moves the 6 away. Latisha also knows that separating the 6 does not make them disappear, but is a distinction that matches the action of the problem: that some are still left (8) and these are part of the 14, but distinct from the 6. As Latisha has demonstrated certain aspects of number knowledge, the teacher can continue to challenge her, using Latisha's own thinking to build sense making. This specific knowledge of Latisha's thinking is instrumental in guiding the instructor to challenge Latisha's conceptual knowledge or misconceptions, while simultaneously displaying the mathematics conversation for the rest of the class. As the teacher gleans knowledge of Latisha's thinking, the teacher's understanding of how word problems and their solutions are structured mathematically is key to moving Latisha further in her conceptual understanding of the mathematics in the discourse. Helping teachers do such critical work has been the focus of CGI research for many years, and researching CGI's framework for PSETs' use is important as they begin their teaching journeys.

Research Questions to Explore Student Thinking

The research questions for this study were formed in part by a gap in research about how PSETs could explore, interpret, and leverage student thinking to build student sense making using mathematical frameworks and pedagogies. Such frameworks and pedagogies could potentially be transferred from PSETs' experiences in mathematics methods courses into their student teaching classrooms. To explore how PSETs could realistically and effectively integrate CGI's framework and body of knowledge into their student teaching classrooms, several core questions needed to be answered. If CGI's elements could be "packaged" in such a way that it would effectively accommodate the unique settings, mathematics lesson objectives, and student needs that PSETs face, important aspects of such a package could be explored. One of the questions that would need to be addressed is observing how PSETs tend to teach problem solving after three semesters have passed since their last use of CGI elements in their methods course. Another question asked how CGI elements could be effectively integrated into PSETs' enacted lessons when working in early number word problems. A third question followed that asked how an intervention with PSETs could be constructed to make CGI pedagogy happen in their placement classrooms. This led to asking what an intervention might look like and how it could be realistically accomplished, accommodating for the already complicated work student teachers are required to do. Since lesson plans are both a standard resource for instruction and a practical tool for helping teachers in their instruction, it seemed prudent to ask how this could be brought about.

As the aforementioned issues coalesced, the questions boiled down to: (1) What elements of CGI do PSETs integrate into early number lesson plans constructed before, during, and after an LPI (lesson plan intervention); (2) What elements of CGI do PSETs enact while they teach early number lessons constructed before, during, and after an LPI; and (3) What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson? To answer these questions, I designed an LPI that would help equip a small group of PSETs to create and enact lesson plans that would reveal student thinking to build their number sense.

A Lesson Plan Intervention to Explore Student Thinking

With the goal of helping PSETs leverage student thinking in early number lessons, this research sought to influence PSETs' focus on attending and interpreting children's intuitive problem solving efforts (Bright, Behr, Post, & Wachsmuth, 1988; Carpenter et al., 1996; Carpenter & Fennema, 1992; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter & Franke, 2004; Jacobs et al., 2010). This study needed a practical way to do this in student teaching placements. Since a number of interventions have successfully assisted student teachers in their placement classrooms, an intervention plan was considered for this study. Hertzog and O'Rode's (2011) intervention investigated the use of mentoring strategies and materials that were designed to support student teachers' use of Mathematical Knowledge for Teaching (MKT). In their work, university supervisors were able to more effectively assess student teachers' MKT use in the field through the use of the field support materials. Gill, Ashton, and Algina's (2004) instructional intervention with preservice teachers showed greater change in implicit epistemological beliefs than the control group, even though it is historically difficult to achieve. Paese's (1984) intervention demonstrated that classroom preservice teaching behavior can be changed through both cooperating teacher interactions and self-assessment techniques. An intervention study of a physical education student teacher revealed positive change using an audio prompting technique (O'Pry & Paese, 1987). An intervention study on science student teachers revealed that "reflective content related knowledge for teaching systems thinking can be promoted in teacher education" (Rosenkranzer, Kramer, Horsch, Schuler, & Rieb, 2016, p. 156). These studies point to the possibilities of using an intervention to bridge PSETs' understanding of CGI framework from their methods course into their student teaching placement classrooms.

With these interventions in view, it seemed appropriate to also use an intervention strategy for PSETs in their student teaching placements. As with many types of pedagogical tools, interventions are often successful if there is an approach that connects theory with practice (Bullock, 2004; Lesh & Doerr, 2003). Since this intervention study needed: (a) specific CGI framework training sessions, (b) specific inputs (CGI teaching practices and problem types/solution strategies), and (c) flexibility to adjust to each of the three PSETs' responsibilities in their placement classrooms, a *lesson plan intervention* (LPI) was used. Lesson plans have long been central to helping teachers successfully plan for and enact lesson-specific goals in classroom instruction (Darling-Hammond, Banks, Zumwalt, Gomez, Gamoran Sherin, Griesdorn, & Finn, 2012). The use of a lesson plan-based intervention helped enact the study's goals of exploring PSETs' pursuits of student thinking. Lesson plans also gave the researcher and PSETs (as lesson plan co-creators) specific ways to integrate CGI practices and problem types into teachable chunks. Specific dialogue and questions were written into the plans, as well as options for how the lesson could leverage student's thinking for other children. Since the lesson plan format used in the study was the standard format used by the PSETs in their methods courses, less stress was involved in writing the lesson plans.

This intervention connected the mathematics methods course with the PSETs' normal teaching responsibilities in their student teaching classroom(s). For the PSETs, the interventions needed to be minimally stressful, with a reasonable amount of extra responsibility. The interventions also needed to integrate CGI elements taught in the methods course into the LPI lesson plans. Additionally, the interventions presented a reasonable chance of helping the PSETs utilize CGI elements in such a way that the students (first graders) would benefit from the experience and not be missing any expected content from the schools' curriculum.

The PSETs demonstrated a willingness to co-create lesson plans based on elements of CGI and to enact these lessons with sincerity and integrity. The study utilized: (a) two PSET self-assessments of CGI framework knowledge and mathematics self-efficacy, (b) two CGI element training sessions to co-create a CGI-based lesson plan (called the LPI lesson plan), (c) CGI framework word problem types and solution strategies, (d) large group mathematics instruction, (e) CGI pedagogical elements, and (f) lesson plan creation.

Part One of the study involved two pre-assessments that were administered to the PSETs. The first assessment asked each PSET to recall their understanding of CGI framework; the second assessment asked each PSET to share their feelings about mathematics and mathematical teaching. These assessments helped establish the PSETs' knowledge of CGI elements and mathematics self-efficacies to see if any relationships existed with successes in attending to, interpreting, and fostering student thinking.

Part Two of the study involved observing each PSET teach a problem solving lesson (called Lesson One) in their student teaching placements. The lesson plan for this first lesson was created and taught solely by the PSET. Immediately after the lesson, an unstructured conversation took place, followed by a semi-structured interview at the end of the day. This gave baseline information for how each PSET taught and which (possible) CGI elements were present in their teaching. It also gave baseline information on how they did or did not elicit student thinking.

Part Three of the study was the lesson plan intervention (called Lesson Two) - it involved two 1+ hour long sessions between each PSET and the researcher to review key elements of CGI and to co-create a CGI-based lesson plan for eliciting and supporting student thinking. Each PSET and the researcher co-created this lesson.

Part Four of the study required each PSET to teach this co-created LPI lesson plan in their placement classroom. The observation of this lesson looked for changes in PSETs' teacher actions and student thinking based on CGI elements. Immediately after the lesson, an unstructured conversation took place, followed by a semi-structured interview at the end of the day.

Part Five of the study required each PSET to create (by themselves) a third lesson plan (called Lesson Three) with story problems and teach it to their first graders. Immediately after this lesson, an unstructured conversation took place, followed by a semi-structured interview at the end of the day. This section of the research looked at how each PSET enacted this third lesson plan without researcher assistance. It was analyzed for the presence/absence of CGI elements, CGI teaching practices, and nuances of elicited student thinking.

It was anticipated that enacted elements of CGI framework would have some effect on PSETs' ability to leverage student thinking, even in fluid learning environments like large group mathematics instruction. The study utilized six data sources: PSETs' mathematics and self-efficacy assessments, lesson observations, lesson plans, conversations, interviews, and analytic memos. Collectively, these provided data about which LPI features and CGI elements helped PSETs leverage student thinking and how instances of student problem solving occurred. The study also looked for similarities and differences in PSETs' actions as well as dichotomies and tensions in their teaching characteristics and dialogs. As a qualitative study, it was also interested in co-occurrences of events and discourses, relationships between lesson discussions and instruction, and language patterns involving CGI practices and student thinking between PSETs and students.

Contributions to mathematics education research came through the distinct structure of this study. Its sections included: (a) use of individualized lesson plan intervention sessions, (b) teaching under a previously unknown cooperating teacher (in two of the three participants' cases), (c) participant and observer role of researcher, (d) co-constructing CGI framework-based lesson plans, (e) conversations and interviews after each lesson, and (f) focus on student thinking and CGI elements in real-time lessons with first graders. The study gave credence to the practical work of student teaching as both an academic requirement and as a place for PSETs to begin their own professional development in mathematical instruction. The study offered understanding into how student teachers implemented CGI elements into lesson plans and enacted them in real time with their first grade students. The research also provided insights into PSETs' teaching practices during student teaching and how they elicited and used student mathematical thinking to build number sense.

Overview of the Study

Sections of the LPI study

Section of Study	Time of Occurrence
Self-assessment of CGI knowledge to collect baseline characteristics	Administered at beginning of the study
Self-assessment of mathematical self-efficacy to collect baseline characteristics	Administered at beginning of the study
PSETs create Lesson One (L #1)	One month after self-assessment
PSETs enact L #1	One month after self-assessment
Conversation with PSETs about L #1	Immediately after enacted L #1
Interview with PSETs about L #1	End of day after enacted L #1
Lesson Plan Intervention (LPI) Session One	One day after L #1
Lesson Plan Intervention (LPI) Session Two	One day after L #1
PSETs enact Lesson Two (L #2)	One or two days after LPI
Conversation with PSETs about L #2	Immediately after enacted L #2
Interview with PSETs about L #2	End of day after enacted L #2
PSETs create Lesson Three (L #3)	One to two days after L #2
PSETs enact L #3	One to two days after L #2
Conversation with PSETs about L #3	Immediately after enacted L #3
Interview with PSETs about L #3	End of day after enacted L #3

Figure 1. Overview of the five sections of the study. The study started with two self-assessments for baseline data about PSETs. Then PSETs created and enacted first lesson, L #1, followed by conversation and interview. LPI sessions followed, resulting in a co-created lesson plan, L #2. PSETs enacted L #2, followed by conversation and interview. PSETs self-created and enacted a third lesson plan, L #3, followed by conversation and interview.

Outline of Thesis

Chapter One articulates the rationale and impetus for this research and presents the research questions. Chapter Two reviews research that is relevant to the study, including: (a) mathematical sense making, (b) student teaching, (c) interventions for preservice teachers, (d) undergraduate teacher education, (e) K-6 student thinking, (f) CGI framework, and (g) lesson planning. Chapter Three articulates the design of the research study, including parts of the study, qualitative methodology, CGI framework integration, participant and placement information, and data gathering and analysis. Chapter Four delineates the findings of the study and ties them back to the research questions. Chapter Five discusses the researcher's interpretation of the findings and what they mean. Chapter Five also discusses conclusions and recommendations for readers interested in lesson plan interventions for student teachers.

Chapter Two: Review of Literature

Our national priorities for mathematics education are displayed in the National Council of Teachers of Mathematics' Principles and Standards for Mathematics (2000) and the Common Core State Standards for Math (National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2010). Common Core State Standards for Math (CCSS-M) include the activities and processes students are doing as they learn mathematics. Students that are regularly challenged by CCSS-M's practice standards will grow to be mathematically *proficient*. Proficient students are able to, among other practices, make sense of problems and persevere in solving them (National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2010). To support student mathematical proficiency, it is imperative that educational stakeholders, including teacher education programs, take action to support children's mathematical proficiency.

This literature review addresses research about: (a) supporting student mathematical proficiency, (b) bridging mathematics methods coursework with student teaching, (c) undergraduate teacher preparation, (d) student teaching, (e) interventions with PSETs, (f) PSETs' lesson planning, (g) mathematical sense making, (h) PSETs and student mathematical thinking, (i) young children's mathematical thinking, (j) CGI, (k) CGI in practice, and (l) CGI limitations.

Bridging Mathematics Methods Coursework with Student Teaching

The priority of supporting student mathematical proficiency is the impetus for this research. In particular, this study addresses students' mathematical proficiency in making sense of problems and persevering in solving them (National Governors'

Association Center for Best Practices and the Council of Chief State School Officers, 2010). Problem solving experiences are a key developmental step in helping children explore the mathematical relationships of objects familiar to them (Carpenter et al., 1996). To help students be successful problem solvers, teacher education programs must equip PSETs (Darling-Hammond, 2010) to successfully engage children in problem solving activities, building children's number sense and conceptual knowledge (Carpenter et al., 2015). Equipping PSETs to do this work is an important part of their preparation as elementary mathematics instructors (Ball, Hill, & Bass, 2005; Shulman, 1987), but often requires PSETs overcoming common challenges (Fehn & Koeppen, 1998). One of these challenges is bridging methods course content and pedagogy into PSETs' student teaching experiences (Fehn & Koeppen, 1998; Valencia et. al., 2009).

This section of the literature review addresses research focusing on the challenges that teacher preparation programs face in preparing PSETs for their student teaching experiences. Teacher preparation programs have the large task of finding out which features best help teacher candidates be successful (Darling-Hammond, 2014). Research on teacher preparation is extensive (Cochran-Smith & Villegas, 2015), covering a myriad of issues, audiences and purposes. Student teachers have the opportunity to enact the theories they have learned in their methods courses while showing their skills and knowledge to their supervisors, cooperating teachers, and students (Stengel & Tom, 1996).

Haefner and Zembal-Saul (2004) report that preservice teachers often do not see models of instruction in their methods courses and/or placement classrooms. The presence of CGI modeling in my mathematics methods course gave the PSETs in this study a foundation for eliciting student mathematical thinking through problem solving. This allowed the LPI to be fabricated with PSETs' knowledge of CGI principles and elements, an important factor in designing the LPI for a reasonable chance of success in bridging CGI prior knowledge into their student teaching placement classrooms.

Darling-Hammond (2010) reported that the clinical side of teacher preparation has little connection to university work. For many reasons, transfer of theory and skills from teacher education programs to K-12 classrooms often does not occur or occurs with irregularity. The bridging of material learned from methods courses into field experiences can be overlooked by all stakeholders. Fehn and Koeppen (1998) investigated student teachers' experiences while implementing methods course material into their student teaching placement classrooms. They discovered that "a social studies methods course can influence how students interpret and behave within the particular teaching situations they encounter" (p. 480).

An important part of teacher preparation programs are the methods courses taken in tandem with field experiences (Darling-Hammond, 2014). Functioning as structured scaffolds, these form the vast majority of scaffolds that teacher education programs provide for preservice teachers. In one study, a scaffolding of preservice teachers to elicit and interpret student mathematical thinking was found to be differentially supportive (Sleep & Boerst, 2012). They revealed that practice-based scaffolds were helpful for beginning teachers and that with some preservice teachers, "additional conceptual and metacognitive scaffolding could have enhanced intern's practice and supported their understanding of the components and rationales for the practice" (Sleep & Boerst, 2012, p. 1046). With this idea in mind, the use of the LPI as a scaffold needed to address a balance of practice-based pedagogies along with the theoretical principles and elements of CGI research in order for the PSETs to have a reasonable chance at implementing CGI practices from their lesson plans to their instruction.

Student Teaching

Research on student teaching is plentiful in many areas, including PSETs' beliefs, practices, and experiences. However, there are fewer investigations into interventions aimed at improving PSETs' elementary mathematics instruction during the student teaching experience. PSETs are uniquely situated in student teaching placements to learn how to move their students to mathematical proficiency. For preservice teachers, student teaching is often the capstone experience to their formal training as new educators (Davenport & Smetana, 2004; Scheeler, McAfee, Ruhl, & Lee, 2006). Learning to teach is a complex task, and teaching programs help preservice teachers prepare for the challenges they face (Darling-Hammond, 2014; Valencia et al., 2009). A common issue in American education is the dichotomy of beliefs and practice, in that teachers were found to endorse the National Council of Teachers of Mathematics standards, but their teaching practices reflected more traditional instructional approaches (Heibert & Stigler, 2004). When cooperating teachers utilize traditional approaches, their examples can be confusing for PSETs, a group least equipped to learn from inferior examples. Darling-Hammond (2014) also stated that dissonance between coursework and fieldwork is a common reality, contravening a cornerstone for teacher education - coherence in coursework and field work. This dissonance is often felt when cooperating teachers try to impress their priorities and models for teaching onto their student teachers.

Edwards and Protheroe (2003) concluded that a "student teacher's learning is heavily situated and that students [student teachers] are not acquiring ways of interpreting learners that are easily transferable, but are learning about curriculum delivery" (p. 227). What these student teachers learned from cooperating teachers about helping individual students, although substantial and useful, did not transfer to helping new students in other placements. Nyaumwe (2004) found that student teachers' conceptions of pedagogy changed more than their conceptions of what it meant to learn mathematics. These student teachers were focused on their improvement of practice for student learning more than student learning itself. Other research offers *framing* – an individual's or group's forming a sense of "what is going on here?"– as a way to help novices attend to student thinking (Levin et al., 2009, p. 146).

Valencia et al. (2009) discussed the loss of student teachers' learning opportunities that arose from competing demands, sparse feedback, and "limited opportunities to develop identities as teachers" (p. 304). Their study also discussed the complexities of student teaching environments that need to be addressed in order for student teachers to be successful. Some of these complexities included relationships between university supervisors, cooperating teachers, and preservice teachers. Schwartz, Walkowiak, Poling, Richardson, and Polly (2018) explored the nature of supervisor feedback given to student teachers about their mathematics lessons. Feedback types varied closely with certain universities, with many variables directly related to the supervisors' connectedness to the programs of the university and their professional development experiences.

Mentoring of student teachers can bridge the gap between educational theory and enacted practice in the classroom. In a study of preservice student teachers, a mentoring relationship with cooperating teachers demonstrated a similarity of beliefs (Zanting, Verloop, & Vermunt, 2001). Mentorship involving preservice teachers takes place at different paces and has many variables affecting transferal of pedagogical styles and priorities. Petancio and Bonotan's (2018) research recommended that "teacher education institutions may consider revisiting their policies and practices to strengthen the support lent by the supervisor, mentors and staff to the budding teachers" (p. 59).

Donche and Van Petegem's (2009) longitudinal study found that third-year teacher education students exhibit more meaning-oriented learning than first-year teacher education students. This suggests that third-year teacher education students sometimes adopt more flexible learning strategies than younger teacher education students.

Supporting student teachers' self-reflections of their teaching performances revealed an increase in the number and depth of their reflective thoughts about content, management, and professional knowledge for teaching (Kong, 2010). Such early growth in student teachers' capacities to learn from their own practices is important for the field of teacher education. Self-reflection about mathematics instruction was enhanced through discourse involving justification and defense of mathematical thinking (Kaminski, 2003). Journaling about their mathematics instruction was an important part of this self-reflective process for student teachers. A focus on reflexive practice with mentors and peers about instructional practices can help preservice teachers learn about teaching (Ferraro, 2000). Zembal-Saul, Krajcik, and Blumenfeld (2002) found that PSETs maintained subject matter emphasis in their classrooms when they were, "provided with opportunities to apply and reflect substantively on their developing considerations for supporting children's science learning" (p. 443).

A part of reflexive praxis is reflecting on one's own position in the classroom. Student teachers have choices about how they perceive themselves in their work. Do they stance themselves (consciously or otherwise) as student teachers, new teachers, pre-professionals or mentees? How student teachers position themselves while student teaching would affect their expectations of themselves and their perceptions of their supervisors' norms for their work. In mathematics instruction this principle can affect a teacher's sense of obligation to students and their positioning as teachers, mentors, mathematicians, etc. (Baldinger & Lai, 2019).

Interventions with PSETs

Hertzog and O'Rode (2011) investigated the use of mentoring strategies and materials that were designed to support student teachers' use of mathematical knowledge for teaching (MKT). University supervisors were able to more effectively assess student teachers' MKT in the field through the use of the field support materials. Osana and Royea's (2011) research used an intervention to improve PSETs' knowledge of fraction concept understanding. Their intervention had mixed results, with some misconceptions being difficult to change or correct.

Interventions for PSETs take many forms and have different degrees of success. Gill et al.'s (2004) research applied an intervention to address preservice teachers' beliefs about mathematics teaching and learning. Their work suggested that an instructional intervention designed to motivate change in specific mathematical beliefs could predict change in preservice teachers' specific epistemological beliefs.

Fennema et al. (1996) state that it is possible for elementary teachers to become skilled at evaluating evidence of student learning through interventions that address beliefs about students' innate ability to problem solve. Spitzer, Phelps, Beyers, Johnson, and Sieminski (2011) revealed that a two lesson intervention helped PSTs more accurately discern student mathematics learning, correctly disregarding irrelevant instances of student actions and statements. However, procedural fluency was still mistaken for conceptual understanding in many instances. Similarly, Philipp (2000) discussed student teachers' beliefs about procedural and conceptual mathematics knowledge for teaching. Many of the student teachers in the study considered procedural knowledge as evidence for conceptual knowledge, a common fallacy for beginning teachers.

Rayner's (2015) intervention study helped PSETs specify learning goals, collect evidence of student learning, and "use the analysis to propose alternative teaching strategies to improve a lesson" (p. iii). Rayner's mathematics methods course intervention utilized teaching video analyses, skill-based instruction, and small group framework development as tools to develop these specific skills. This study documented the successes of using videos and video analyses of lessons in creating specific teaching strategies and learning goals, both of which would be helpful in an LPI that included the elements of CGI practices. Since a CGI-research-based LPI for PSETs would necessitate training in specific content (problem types and accompanying solution strategies), Rayner's use of video analyses could be useful in this LPI study.

PSETs' Lesson Planning

Morris and Hiebert's (2017) study revealed that PSETs' content knowledge in their teacher preparation program was related to their performance on writing lesson plans two to three years after they graduated. Results hinted that the teachers could write better lesson plans if they had spent a fair amount of time on specific subjects in their methods classes. The implication is that increased content knowledge influenced PSETs' lesson planning outcomes. Taylan (2016) explored whether PSET lesson analysis skills during a teacher education course had any effect on their ability to write lesson plans. "The PSETs' lesson analysis scores were significantly and positively correlated with scores in lesson planning tasks focusing on student thinking" (Taylan, 2016, unpaginated).

Ozogul, Olina, and Sullivan (2008) reported greater improvement in preservice teachers' written lesson plans assessed by instructors than preservice teachers' written lesson plans assessed by preservice teachers' peers or by preservice teachers themselves. This had implications for how teacher education programs help preservice teachers create, edit, and assess their lesson plans that align with stated outcomes. In this intervention, this related to how the researcher might help PSETs build their LPI lesson plans to elicit and utilize student thinking.

Drost and Levine's (2015) exploration of 87 preservice teachers' lesson plans showed their preferences for three different instructional approaches (expository, hands-on, and collaborative) and a focus on completion of their lesson plans more than on student engagement. Lesson plans are a pivotal tool for teachers to envision how content is delivered and how pedagogical tools are practiced (Clark & Yinger, 1987). Lesson planning affects what PSETs enact in their classrooms and are markers for what they have learned and valued. We know this is important and is often difficult for them to practice and should be addressed in this forthcoming study.

Mathematical Sense Making

Mathematical sense making is a key aspect of children's learning where mathematical concepts, numerical relationships, and mathematical reasoning come to the mind of a child (National Council of Teachers of Mathematics, 2014). Mathematical sense making happens when students engage with familiar mathematical concepts in new situations. Often, sense making involves exploring representations that are manipulated by the child to answer a specific question or scenario presented to them. Sense making is "trying to make sense of the mathematics under consideration" (Van Zoest et al., 2017, p. 34). The importance of sense making, also called meaning making, is evidenced in the mathematical practices of reasoning abstractly, constructing viable arguments, modeling, and critiquing the reasoning of others (National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2010). When students are able to manipulate strategies through adaptive reasoning, they are demonstrating sense making. Students who are equipped for sense making show conceptual understanding through flexible problem solving and logical thinking and can justify their reasoning (National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2010). The National Governors' Association Center for Best Practices and the Council of Chief State School Officers (2010) standards promote students' meaning making, where mathematical concepts connect with prior knowledge to build new knowledge. Since sense making is a key goal for PSETs as they teach mathematics in their student teaching classrooms, it is important that the LPI under consideration keeps sense making as a key goal for the PSETs as they write their lesson plans.

"The role of the teacher during whole-class discussions is to develop and then build on the personal and collective sense making of students" (Stein, Engle, Smith, & Hughes, 2008, p. 315). Schoenfeld (1992) described mathematical sense making as a kind of flexible understanding that helps students focus on conceptual understandings and not just procedures. Mathematical sense making occurs when students are engaged with numbers on a conceptual level. Curriculum and pedagogy that emphasize conceptual approaches to learning can build sense making through guided inquiries that link previous knowledge in new ways. Leatham et al. (2015) recognized the importance of noticing potentially productive student thinking, then following a sequence where student thinking is carefully put on display for other students to examine (Smith & Stein, 2011).

PSETs and Student Mathematical Thinking

There is evidence that preservice teachers can attend to student thinking "when their professional environment emphasize and encourage novices with regards to paying attention to student thinking" (Taylan, 2016, p. 3). Student thinking includes making sense of word problems, constructing mathematical chains of thought, critiquing mathematical reasoning, and receiving feedback that is "descriptive and timely" (National Council of Teachers of Mathematics, 2014, p. 9). We know attending to student thinking is important (Franke & Kazemi, 2001) and is hard for PSETs to practice. The LPI of this study will need to help PSETs effectively use CGI practices when probing student thinking in their large group instruction.

It is imperative that teachers craft lessons that elicit, interpret, and support student thinking (Carpenter & Fennema, 1992). Pursuing student thinking while teaching enables students to "build on procedural fluency from conceptual understanding", moving past algorithmic patterns to problem solving through experimenting with mathematical concepts tasks (National Council of Teachers of Mathematics, 2014, p. 10). It is important that teachers craft lessons that elicit and support student thinking (Carpenter & Fennema, 1992).

We know from Hughes (2007) that interventions for preservice teachers from mentor teachers and university supervisors "may be an important factor in determining whether or not the teacher applies their knowledge of attention to students' thinking to their planning in practice" (p. v). We know that mentors and instructors must help PSETs build a high capacity to attend to student ideas (Barnhardt & van Es, 2015) to successfully help students progress in their student teaching. This has implications for this study, that my assistance in helping the PSETs attend to student thinking is a difficult skill and should be addressed in the training involved in the LPI.

Schack, Fisher, Thomas, Eisenhardt, Tassell, and Yoder (2013) found that a teaching module using video excerpts increased PSETs' professional noticing of student mathematical thinking. The PSETs demonstrated significant growth in attending, interpreting, and making decisions while teaching young children mathematics. Philipp, Ambrose, Lamb, Sowder, Schappelle, & Sowder (2007) concluded that PSETs "developed more sophisticated beliefs about mathematics, teaching, and learning and improved their mathematical content knowledge" (p. 438) when concurrently learning about children's mathematical thinking while watching videos of children solving problems. As a practical and effective tool for PSETs' education, video analyses of student problem solving sessions would be an efficient way to communicate real life vignettes of student-teacher interactions in this LPI.

Oonk, Verloop, and Gravemeijer (2015) analyzed student teachers' use of educational theory in their placement schools. They concluded that student teachers can develop their own theoretical perspectives through analysis, description, and discussion of their own (and others') real teaching practices. Student teacher self-analyses can be affected by their pedagogical content knowledge (PCK), but PCK is not always enough for teachers to be successful (Silverman & Thompson, 2008). A study of two PSETs revealed that they used their limited PCK to anticipate and reflect on problem events during instruction, but their limited PCK and lack of confidence hindered these processes (Roth McDuffie, 2004).This process continues over time as reflection takes place. As it is common for preservice teachers to have limited PCK and for this to affect their ability to reflect on student thinking in the real-time environment of large group instruction, the study could answer some questions about PSETs' PCK and how it would affect their use of CGI elements to elicit and interpret student mathematical responses and thinking.

Boerst, Sleep, Ball, and Bass (2011) discussed foundational elements for PSETs' successful leading of whole-class mathematics discussions. Their results suggest that teaching practices should be decomposed into smaller parts (nesting), generating approximations of practice. Combined with supportive assessment, nesting then allows for specific subject matter to be taught effectively. Van Zoest et al.'s (2017) investigation unveiled specific attributes of mathematical discourses that often lead to "opportunities to modify instruction in order to extend or change the nature of students' mathematical understanding" (p. 3). Since there are these attributes of mathematics discussions that lead to student mathematical growth, this study could shed light on instances of successes or failures of PSETs' instructional practices that led to student mathematical breakthroughs.

Crespo (2000) reported that interpreting students' mathematical thinking is a difficult skill for new teachers, who often focus on correct or incorrect answers when assessing student mathematical work. Schack et al. (2013) note that PSETs trained to professionally notice children's early numeracy demonstrated significant growth in their capacities to attend, interpret, and make decisions about children's mathematical thinking. However, the skill of noticing student thinking (Barnhardt & van Es, 2015) was shown to not necessarily be effective in analyzing the content of students' mathematical responses. Helping the PSETs in this study will require the LPI to go beyond the surface of students' responses, using CGI's framework of analyzing early mathematics thinking to help them with the difficulties of connecting students' solution strategies with the mathematical concepts underpinning the strategies.

Darling-Hammond (2010) examined the prevalence of algorithmic-centered fact memorization approaches to mathematical instruction in the United States and discovered that these approaches are counter to more effective kinds of instruction found in nations where the focus is on solving problems and exploring conceptual ideas in mathematics. "Math problems should encourage and acknowledge the different ways in which people see mathematics and the different pathways they take to solve problems. When these changes happen, students engage with math more deeply and well" (Boaler, 2016, p. xii). For example, when curriculum is designed to guide students' explorations with mathematics problems, the teacher is better situated to help students make conceptual connections not readily visible from more procedurally based curriculum. Curriculum that emphasized conceptual understanding of fractions through the use of multiple modes of representations was shown to affect elementary students' likelihood that they would approach such problems conceptually (Cramer, Post, & DelMas, 2002). We know that conceptual understanding is important for students and that the PSETs in this study will have varied conceptual understandings. The LPI study could help answer how the PSETs' stance about teaching for conceptual knowledge affects their integration of CGI elements when eliciting and interpreting children's responses.

Cognitively Guided Instruction

CGI is a mathematical framework for teachers that explains children's mathematical thinking about basic number concepts and operations (Carpenter et al., 1989; Carpenter & Fennema, 1992; Fennema, Franke, Carpenter, & Carey, 1993). CGI is also an instructional model that emphasizes teachers' leveraging of students' mathematical thinking to help students learn foundational mathematical concepts and solve problems (Fennema et al., 1993). However, CGI is not a curriculum. CGI classifies word problem types within the four operations and lays out common solution strategies for each type. Each solution strategy is categorized by the type of word problem it solves and by the level of mathematical sophistication a child demonstrates that uses this strategy. With this knowledge of children's thinking, a teacher is equipped to help children explore their offered strategy in a way that unpacks the concepts behind their strategy (Carpenter et al., 2015).

CGI research has shown that children have real life experiences with quantities and they also possess a natural, intuitive sense of how to solve beginning mathematics problems (Carpenter et al., 2015). CGI research has also shown that students' solution strategies are legitimate places to start mathematical conversations with children as they solve problems (Fennema et al., 1993). With CGI's framework for understanding children's mathematical thinking, a CGI practitioner guides students (through specific and general questioning) to explore and extend children's solution strategies, helping children explore the mathematical concepts that underlie their strategies. As a CGI practitioner guides students through their solution strategies, opportunities arise to compare strategies and extend students' early number knowledge. CGI also recognizes that children's mathematical understandings grow over time, becoming more mathematically sophisticated as they have opportunities to explore the relationships between numbers and operations (Fennema et al., 1993; National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2010). The CGI framework helps teachers honor students' thinking, starting with what children know about basic number concepts and operations, and supporting students as they use their own intuitive problem solving strategies. Children's interactions with objects give students opportunities to learn basic counting strategies, which vary from child to child based on their own intuitive abilities (Carpenter & Fennema, 1992). As students solve problems, they use their experiences with common objects to subitize, count, and sort.

Effective teaching models are important because they empower PK-12 students to be mathematically tenacious, skilled, creative problem solvers and thinkers (Carpenter, Fennema, Franke, Empson, & Levi, 1999). CGI research focuses on how "teachers use research-based knowledge about children's thinking and problem solving to make decisions as they plan and implement instruction, and how this instruction affects their students' learning” (Carpenter & Fennema, 1992, p. 458). CGI research focuses on understanding children’s mathematical thinking, especially of counting and problem solving. As children learn to count, they gather skills that help them understand important mathematical concepts like one to one correspondence and ordinality. These emerging counting skills lay the foundation for learning to solve word/story problems (Carpenter et al., 1999). CGI is an excellent framework to leverage student thinking, with specific elements that help teachers recognize nuances of problem types and their difficulties for students. CGI practices also gives teachers specific details about children’s solution strategies and how these strategies are used by children as they solve word problems. These CGI strengths are effectively utilized for instruction, making CGI an excellent framework for use in the LPI. Because the CGI framework values and fosters student thinking and emphasizes intentional listening and interpreting of student verbalizations (Jacobs et al., 2010) it serves as a stage for teaching in large groups.

Problem solving activities that encourage high level cognitive reasoning can result in children who “become competent and confident in their ability to tackle difficult problems and willing to persevere when tasks are challenging” (National Council of Teachers of Mathematics, 2014, p. 2). Building on children’s capacity to problem solve is a difficult and important task for teachers (Carpenter et al., 2015; National Governors’ Association Center for Best Practices and the Council of Chief State School Officers, 2010). For this reason, an LPI was designed to help PSETs lead children in problem solving activities, building children’s number sense and concept understanding.

CGI is “a stance that looks for what children know, and centers on children’s ideas and sense making in the teaching and learning of mathematics” (Carpenter et al., 2017, p. 5). CGI is not a curriculum or pedagogy, but attempts to guide teachers through problems as children would see them, and then leveraging this understanding to help students make conceptual connections. “It turns out that young children are remarkably successful in intuitively applying their counting skills to solve a variety of problems in a number of different contests” (Carpenter et al., 2017, p. 4). When children’s conceptions and misconceptions are uncovered, a PSET can ask direct questions of students about how they solved the problem and help students reflect on their thinking (Fennema et al., 1993).

CGI theory recognizes that “children start school with a conception of basic mathematics that is much richer and more integrated than that presented in most traditional mathematics programs” (Fennema, Carpenter, Franke, & Carey, 1992, p. 4). Children’s life experiences with objects and quantities form a foundation for learning how to solve simple problems. Sharing toys, counting game pieces, talking with older children and adults - all supply children with real-life mathematical experiences. The skill of counting forms a basis for further mathematical learning and success (Carpenter & Fennema, 1992; Stock et al., 2009). The CGI model utilizes these life experiences by acknowledging children’s use of invented strategies as important ventures in children’s progression to learning mathematics. PSETs who learned CGI research and practices in their mathematics methods courses begin to recognize and value children’s conceptual and procedural understandings. CGI research and practices emphasize that teachers make reflection time available for children to think about mathematics through discussion, questioning, experimenting with manipulatives, and sharing their thinking with teachers and other students.

CGI in Practice

CGI researchers see "time and again that the details of children's mathematical ideas matter, and that noticing and building from children's thinking creates powerful learning opportunities" (Carpenter et al., 2017, p. 3). Hendricks' (2013) study of 104 second graders revealed "a significant difference between math scores from the treatment [CGI] group" (p. 3). Medrano (2012) studied second through fourth graders and concluded that students benefited from CGI instruction. A study of 21 Scottish in-service teachers using CGI practices found that teachers' "increased understanding of children's mathematical thinking left them better placed to support all learners" (Moscardini, 2014, p. 2). Research by Moscardini (2014) discovered that "all the participating teachers considered themselves to be more knowledgeable about children's mathematical thinking" (p. 18). Baker and Harter (2015) equate CGI framework as an instructional model similar to differentiated instruction. Their research of six studies by Carpenter and colleagues pointed out that "CGI practices could offer an optimal pedagogical approach for students of different demographics because the framework uses each student's unique background and knowledge to inform teaching" (Baker & Harter, 2015, p. 34).

A longitudinal study of 21 teachers' instructional beliefs revealed that, "Over the four years, there were fundamental changes in the beliefs and instruction of 18 teachers such that the teachers' role evolved from demonstrating procedures to helping children build on their mathematical thinking by engaging them in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking"

(Fennema et al., 1996, p. 403). A study of CGI practices concluded that as teachers gained experience in the use of CGI practices with colleagues, they became a community of learners and saw their classrooms as places for experimentation for their own learning and professional growth (Franke and Kazemi, 2001).

Roche (2013) discussed the difficulties and common practices of in-service elementary teachers when choosing, creating, and using story problems. Using CGI framework to explain the structure of word problems, she suggested five criteria for presenting and using word problems that clarify the contextual and mathematical nuances important for conceptual understanding. Training with CGI principles is often accomplished through professional development programs. Implementation of CGI principles through professional development programs can be effective when administration is involved beyond a cursory level (Guerrero, 2014). In-depth instruction for teachers was also a necessary component for teachers to successfully use CGI practices in their classrooms, with a lack therein reported as a hindrance in the three schools in the study.

CGI framework directly addresses many important foundations for student mathematics learning, including focusing on students' thinking and valuing teaching that focuses on concepts, discussions, and meaning making. CGI framework addresses key ideas espoused by current mathematical education research. CGI framework is one way of following this kind of mathematical instructional philosophy, with its focus on conceptual approaches to mathematics instruction, welcoming and exploring children's mistakes, and emphasizing that teachers need to carefully listen to children's mathematical thinking and solution strategies (Carpenter et al., 2015). When children's conceptions and misconceptions are uncovered, a PSET can ask direct questions of students about how they solved the problem and help students reflect on their thinking (Fennema et al., 1993).

CGI practitioners foster classrooms where "each teacher creates a teaching and learning environment that is structured to fit his or her teaching style, knowledge, beliefs, and children" (Fennema et al., 1992, p. 2). In CGI lessons, children investigate problems using various mathematical manipulatives to help them find multiple solutions. Children very often use abstractions of the physical manipulations they initially used to solve the problems (Carpenter & Levi, 2000). Each child is free to solve each problem by themselves, or with a partner, from their own intuition. The teacher guides students as they work, using content knowledge, child-centered pedagogy, and the CGI framework (Fennema et al., 1992). Students are given ownership of their tasks, respected as thinkers, and encouraged to share their thinking with others.

Carpenter et al.'s (1989) research showed "that teachers' knowledge and beliefs about students' thinking are related to students' achievement" (p. 457). Classes where CGI principles were used showed differences in freedom given to students to solve problems, teachers' knowledge of students' thinking, and student achievement in problem solving skills. The CGI model fosters student exploration, giving them a mathematical mindset (Boaler, 2015) that is needed for further learning.

Limitations to CGI

CGI authors found that the model had not directly increased computational efficiency, but did not hurt it either (Carpenter et al., 2015; Carpenter & Fennema, 1992). While computational efficiency is important, conceptual understanding is what allows students to solve non-standard problems creatively and accurately (Schoenfeld, 1992). CGI's focus on supporting student thinking does not preclude hard work on both the teacher's and the students' parts. Vacc and Bright's (1999) investigation showed that preservice teachers engaged in a semester course in CGI instruction showed changed beliefs about mathematics instruction, but were unable to use this in their teaching. Franke, Webb, Chan, Ing, Freund, & Battey, (2009) discovered that in-service teachers who had experienced CGI training through professional development showed variability in asking follow up questions to elicit student thinking.

Curriculum factors also play a role in CGI implementation. Since CGI is not a curriculum, teachers who wish to use CGI principles must integrate CGI elements into whatever curriculum the school uses. Although Heinemann publishes many excellent CGI resources for teachers (Carpenter et al., 2015), it takes time for PSETs to learn how to utilize such resources. Experience is required to understand and implement the pedagogical aspects of posing questions and waiting for answers. Some commercially produced mathematics curricula are not centered on student thinking, but on algorithms and specific strategies to solve problems. However, some curricula are more student thinking centered and are good fits for implementing CGI tenets in the classroom. These types of curricula more easily lend themselves to CGI's framework for instruction. With CGI framework emphasis on leveraging student discussions to illuminate problem solving strategies, teachers need training to see how students are able to solve problems intuitively and that the sharing of student strategies is valuable and pedagogically effective.

Other factors also play a role in CGI practice effectiveness. These include teachers' knowledge of early number concepts and the steps that students often move through as they learn early number concepts and problem solving skills. Classrooms where student-student conversations are fostered is also a challenge for many, philosophically and managerially, but changes are possible when knowledge of how children think increases (Fennema et al., 1996). It takes time for teachers to see student thinking in real time, assess what the student(s) knows about that particular concept, make a pedagogical decision to elicit a strategy for solving the problem, and finally orchestrate classroom (or small group) discussions around other students' thinking - all in real time (Peterson & Leatham, 2009). This is a challenging skill set for any teacher wishing to understand the value of leveraging student thinking.

Summary of the Research

This chapter reviewed areas of research pertinent to the design of this study - helping PSETs teaching in their student teaching classrooms to support their students during problem solving activities. This research articulated ideas that led to building a CGI-based LPI for PSETs. Research was reviewed about the importance of helping teachers support student mathematical proficiency. Other research articulated the difficulties and opportunities of bridging mathematics methods coursework with student teaching. Undergraduate teacher preparation research was reviewed that iterated the importance and challenges of helping teacher education programs equip PSETs to understand content and pedagogy related to mathematical instruction. Student teaching research was discussed that revealed opportunities PSETs have to support young children's mathematical thinking and the hurdles PSETs must overcome to make this happen. Research about interventions that supported PSETs' efforts to successfully learn and apply instructional models was reviewed. A check of research articles occurred about PSETs' lesson planning practices and supports, giving insights about how the LPI could help PSETs construct lesson plans that integrated CGI tenets for eliciting and supporting student problem solving activities. Research related to student mathematical sense making was explored that laid a foundation for helping build an LPI that would equip PSETs to leverage CGI's framework of student mathematical thinking while problem solving. Other studies reviewed ideas about how young children think mathematically and how their strategies for solving problems could be analyzed through CGI framework. More research was mentioned that connected children's mathematical thinking that could be explored through an LPI's use of lesson plans to integrate elements of CGI. Studies about CGI practices were discussed that illuminated CGI's fit for forwarding mathematical sense making through eliciting and utilizing student thinking during instruction.

Chapter Three: Research Methods

Research Design

The purpose of this research was to explore whether a CGI framework-based lesson plan intervention (LPI) would help preservice elementary teachers (PSETs) leverage student thinking to increase mathematical sense making during large group instruction. An aim of this study is to implement a direct application of learning theory to "improve the practice of a particular discipline" (Merriam, 2009, p. 4). This intervention study explored the influence of two CGI framework-based lesson planning sessions on student teachers' instruction in early number topics. It also examined the interactions of PSETs with their students as they sought to understand, interpret, and utilize student thinking during large group instruction. The LPI attempted to help the preservice teachers recall CGI knowledge that PSETs had learned in their mathematics methods course taken three semesters previously. As a multiple case study, it provided data for each PSET as well as comparative data among PSETs. Multiple case studies bound the same types of data across cases and are contained within a common broader context.

There are multiple components to this research that occurred three semesters after participants took their elementary mathematics methods course taught by the researcher. First, it looked at what a CGI framework-based LPI could do for a PSET's mathematics instruction. Second, it explored aspects of CGI practices that PSETs use, don't use, or partially use when planning and teaching early number lessons. Third, this research examined what particular aspects of CGI elements might be easier or harder to integrate into their teaching practices. Fourth, the research investigated PSETs' teaching practices that existed or emerged before, during, and after the CGI framework-based lesson plan interventions in early number. Finally, the study analyzed student mathematical thinking that was elicited or expanded upon through PSETs' use of the lesson plans they created.

Chapter Three articulates all the components of the research design including its qualitative methodology, LPI design, CGI framework and mathematical content, structure of CGI framework-based lesson plans, and participants in classroom contexts. A breakdown of the five parts of the LPI follows, including details of what the LPI offered the PSETs as well as how it was actualized. The chapter finishes with details of data source coding, cycles of coding and analyses, and trustworthiness of the study.

Research Questions

The three research questions were: (1) "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?", (2) "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?", and (3) "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" An additional set of three research questions (in Appendix H) were a part of the original design. They addressed issues of student mathematical learning. These three questions (four through six) were dropped from the study when it became evident that there was insufficient data for analysis. Determination of this decision occurred after the first and second cycles of lesson observations and post-lesson conversations and interviews.

Qualitative Methodology

To answer these questions, this study implemented a multi-case, qualitative methodology through a lesson plan intervention (LPI) designed to help participating preservice elementary teachers (PSETs) leverage student thinking to increase student sense making of mathematical ideas during large group instruction. The three PSET participants represented a wide spectrum in cognitively guided instruction (CGI) knowledge, teaching self-efficacy, and teaching experiences. To accommodate for these differences while drawing out the full effects of an LPI, a multi-case study methodology was chosen. “Multicase field research is very useful in providing contrast and variance” (Miles et al., 2014, p. 292). Cohen et al. (2011) advocate that research should address authenticity and fitness for purpose. The multi-case design fits the purposes of this research – it illuminated the PSETs’ experiences as they used CGI framework-based lesson plans to leverage student thinking to build number sense. The multi-case approach looked for similarities and differences in PSETs’ implementation of lesson plans as they taught problem solving to their first grade students. “Case studies can represent something of the discrepancies or conflicts between the viewpoints held by participants” (Cohen et al., 2011, p. 292).

This research applied the teaching framework of CGI to the direct practices of student teachers engaged in mathematics instruction with first graders in public schools, using some tenets of applied research (Miles et al., 2014). The plan of the research was practical in nature, not attempting to document the validity of CGI research, but to leverage the strengths of CGI practices in helping novice teachers recognize and leverage student mathematical thinking to build number sense (Van Zoest et al., 2017). The study was interested in the practicality and usefulness of using an LPI to leverage student thinking, and qualitative research can often accomplish this (Patton, 2015). The use of case study methodology enabled careful observation and analysis of beginning practitioners in real time, supporting transparency and clarity in the observations, interviews and conversations with the participants. Case study methodology gave room for open-ended questions, depth of participant responses, and exploration of participants' decisions and practices when teaching mathematics problem solving (Patton, 2015).

Components of the LPI Study

Component	Time Table
Self-assessment of CGI knowledge to collect baseline characteristics	Administered at beginning of the study
Self-assessment of mathematical self-efficacy to collect baseline characteristics	Administered at beginning of the study
PSETs create Lesson One (L #1)	One month after self-assessment
PSETs enact L #1	One month after self-assessment
Conversation with PSETs about L #1	Immediately after enacted L #1
Interview with PSETs about L #1	End of day after enacted L #1
Lesson Plan Intervention (LPI) Session One	One day after L #1
Lesson Plan Intervention (LPI) Session Two	One day after L #1
PSETs enact Lesson Two (L #2)	One or two days after LPI
Conversation with PSETs about L #2	Immediately after enacted L #2
Interview with PSETs about L #2	End of day after enacted L #2
PSETs create Lesson Three (L #3)	One to two days after L #2
PSETs enact L #3	One to two days after L #2
Conversation with PSETs about L #3	Immediately after enacted L #3
Interview with PSETs about L #3	End of day after enacted L #3

Figure 2. Overview of the sections of the study. The study started with two self-assessments for baseline data of PSETs. Then PSETs created and enacted first lesson, L#1, followed by conversation and interview. LPI sessions followed, resulting in co-created lesson plan, L#2. PSETs enacted L#2, followed by conversation and interview. PSETs self-created and enacted third lesson plan, L#3, followed by conversation and interview.

Steps of Research

To support and enhance PSETs' use of CGI practices in their lesson planning and teaching, an LPI was constructed that utilized CGI material from PSETs' previous mathematics methods course. First, two self-assessments were created and individually administered to understand each PSET's knowledge of CGI framework and mathematical self-efficacy. One self-assessment asked PSETs to share their recalled knowledge and reflections of CGI framework. The assessment was completed privately by the PSETs. The second self-assessment asked participants about their experiences doing mathematics and their reflections about teaching mathematics to children. Participants were identified and admitted to the study if they agreed to participate in the study, if they were teaching in a primary grade classroom, and if they completed the self-assessments. Four PSETs met these criteria and were admitted to the study. One student left her student teaching placement and did not complete the study. After these self-assessments, the PSETs were asked to create and teach an early number lesson plan with word/story problems. Video recordings of these first lessons established a baseline of PSETs' practices in their classrooms of first graders. Post-lesson conversations and semi-structured interviews followed each first lesson. One to two days later, each PSET and I engaged in two LPI training sessions. The first session reviewed the elements of CGI framework and included analysis of videos of students solving problems as well as analysis of written examples of children's problem solving work. The second session of the LPI occurred to co-create a lesson plan in early number, integrating CGI elements so that PSETs could elicit and leverage student strategies to solve word problems. This second lesson was observed and video recorded, then followed up with a conversation and an interview. From there each

PSET individually wrote and taught a third lesson plan, again followed by a conversation and interview to gather data. Each PSET wrote their own third lesson plan involving word problems. Analysis of their third lesson plans and enacted lessons looked for instances of CGI integration. Analysis of lesson plans, enacted lessons, conversations, and interviews was conducted using a constant comparative framework.

Participants

This research involved PSETs from a small, Midwestern, private college. Potential PSETs came from a pool of 10 student teachers that completed their student teaching in public and private schools during the spring semester of their fourth and final year of Baccalaureate work. The pool of 10 teachers had successfully completed all of their degree coursework. They also had successfully completed the researcher's elementary mathematics methods course a year and a half previously. All of the PSETs had many experiences evaluating students' thinking in the mathematics methods course and observed how students solved the mathematics problems presented to them. During the course, some PSETs demonstrated robust knowledge of CGI practices, others less so.

Mathematics Methods Course

Concurrent with the mathematics methods course, the PSETs were in practicum experiences in school classrooms all day on Tuesdays and Thursdays. The mathematics methods course included an assignment where they had to interview three students about a mathematics problem and give their analysis of student thinking, possible misconceptions, and solution strategies. In the elementary mathematics methods course, PSETs were also presented with various strategies (in CGI framework) that children use to solve problems and the types of problems that these strategies help solve (Appendices

E and F). In the mathematics methods course, PSETs learned about other elementary content areas including place value, whole number operations, fractions and decimals, and beginning algebra.

Participant Selection

Of the 10 potential PSET participants, five were placed in intermediate grade classrooms and so were not candidates for this study, as this research was focused on early number sense making. Of the remaining five, one student changed his/her mind about being in the study, leaving four participants who were in first or second grade classrooms. These four remaining students consented to the research. One month into the student teaching placements, one of the participants became ill and had to relinquish her placement. This left three participants in this study: Jennie, Penny, and Eleanor. All names are pseudonyms.

All three full participants were female. For all three PSETs, the researcher was not their student teaching supervisor and did not have any direct responsibility for their student teaching work. This allowed for a neutral stance with the participants and negated any potential conflicts of interest where the author would have been both their supervisor and researcher. Care was taken to make clear that the three lesson observations, lesson plans, interviews, and conversations would not be for evaluation or grading purposes. A clear statement of consent was followed, in accordance with the University's IRB protocol.

School and Classroom Context

Three first grade classrooms from three different public schools in the Midwest were hosts to the three PSETs. All three schools were third ring suburbs of a large metropolitan area and had two or three ELL learners in each classroom. Class sizes for the schools averaged 24 students and PSETs' classrooms were 22, 23, and 25 students each. For all three PSETs, mathematical instruction was with students from multiple first grade classes, arranged in flexible groups based on test scores. All three cooperating teachers for these PSETs grouped their students on unit pre-test scores, given about once a month. This meant each PSET had all first grade students, not solely students from their home classrooms.

PSET Jennie described her mathematics group with, "All of the students in this math class struggle with math so we are attempting to meet their needs by making our math very concrete and reaching all our various learnings styles by making math very hands-on, active, visual, and doing constant think alouds in order to show students how math thinking works". Jennie usually had 13 students in her large group, observed lessons.

PSET Penny described her mathematics group with, "This class is very high needs. Multiple students are on IEPs. Two special education teachers push in for half of the [math] class time and the other half paraprofessionals join the room". Penny usually had 16 students in her large group, observed lessons.

PSET Eleanor described her group as a "high group", according to the latest mathematics content exam, given once a month. In one lesson plan she wrote, "One student has EBD and is in the process of evaluation for services. He will need help staying on task. One student has autism spectrum and has a paraprofessional. One additional student needs help staying on task." She had 26 or 27 students involved in all three of her observed lessons.

All three lessons for each PSET were with the same mathematics group, using whatever curriculum the school was using. All three participants' schools used Houghton Mifflin Harcourt's (2018) Math Expressions curriculum.

Summary Chart of Structure of Data Collection

Research questions and data collection.

Research Question	Data Source	Description
1. What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?	Lesson Plan One	Analysis of pre-LPI lesson plan revealed baseline of PSETs' integration or non-integration of CGI elements
	Lesson Plan Two	Analysis of LPI lesson plan revealed PSETs' integration/non-integration of CGI elements
	Lesson Plan Three	Analysis of post-LPI lesson plan revealed PSETs' integration/non-integration of CGI elements
2. What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?	Enacted Lesson One	Audio and video recorded observation of pre-LPI lesson; looked for elements of CGI present or not present in instruction
	Lesson One Conversation	Audio recording of *unstructured* conversation immediately after the pre-LPI lesson; looked for participant thoughts and intentions about CGI elements
	Lesson One interview	Audio recording of *semi-structured* interview after the pre- LPI lesson; looked for more specific details about participant thoughts and intentions about enacted CGI elements
	Analytic Memos	Analytic *memos* recorded researcher's impressions and ideas about CGI elements present or not present in instruction; *memos* also recorded impressions from conversation and interview with PSETs

	Enacted Lesson Two	Audio and video recorded *observation* of LPI lesson; looked for elements of CGI present or not present in instruction
	Lesson Two Conversation	Audio recording of *unstructured* conversation immediately after the LPI lesson; looked for participant thoughts and intentions about CGI elements
	Lesson Two Interview	Audio recording of *semi-structured* interview after the LPI lesson; looked for more specific details about participant thoughts and intentions about enacted CGI elements
	Analytic Memos	Analytic *memos* recorded researcher's impressions and ideas about CGI elements present or not present in instruction. *Memos* also recorded impressions from conversations and interviews with PSETs
	Lesson Three	Audio and video recorded *observation* of post LPI lesson; looked for elements of CGI practices present or not present in instruction
	Lesson Three Conversation	Audio recording of *unstructured* conversation immediately after the post LPI lesson; looked for participant thoughts and intentions about CGI elements
	Lesson Three Interview	Audio recording of *semi-structured* interview after the post LPI lesson; looked for more specific details about participant thoughts and intentions about enacted CGI elements

	Analytic Memos	Analytic *memos* recorded researcher's impressions and ideas about CGI elements present or not present in instruction. *Memos* also recorded impressions from conversations and interviews with PSETs
3. What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?	Enacted Lesson One	Audio and video recorded observation of pre-LPI lesson; looked for CGI framework teaching practices present or not present in instruction
	Enacted Lesson Two	Audio and video recorded observation of LPI lesson; looked for CGI framework teaching practices present or not present in instruction
	Enacted Lesson Three	Audio and video recorded observation of post LPI lesson; looked for CGI framework teaching practices present or not present in instruction
	Analytic Memos	Analytic *memos* recorded researcher's impressions and ideas about CGI framework teaching practices present or not present in all three lessons. *Memos* also recorded researcher's impressions from conversations and interviews with PSETs

Figure 3. Research question, data source, and description of data source collection for all six research questions. Included are some details about how the data source helped answer the research question.

Two Pre-LPI Assessments, CGI Framework, and Self-Efficacy

To find baseline information about PSETs' CGI practices and mathematics self-efficacy, two assessment tools were developed and administered. A CGI framework assessment sought participants' knowledge of tenets of CGI framework, and the second assessment (mathematics self-efficacy) sought participants' self-efficacy in their mathematical abilities and experiences as well as their feelings about teaching mathematics in their student teaching placements. The CGI framework assessment asked participants about the major elements of CGI framework: knowledge of mathematical problem types, solution strategies, asking follow up questions, and valuing student thinking. One of the purposes of the CGI framework assessment was to determine which PSETs would remember CGI material from the elementary methods course taken three semesters previously. The second assessment (mathematics self-efficacy) asked participants to share their feelings about mathematics, their previous experiences with mathematics teachers, and their feelings about teaching mathematics in their student teaching placement classrooms.

CGI framework knowledge assessment. The Cognitively Guided Instruction knowledge assessment was given to 10 potential participants one time, before their student teaching placements began. Potential PSETs took the assessment individually, with no time constraints given. The 13-question assessment (Appendix B) covered elements of CGI, all of which were taught in the prior mathematics methods course. The assessment's purpose was to determine participants' working knowledge of CGI elements before the CGI framework-based lesson plan intervention took place.

CGI knowledge assessment design. The content of the CGI practices assessment was based on selected CGI elements emphasized in the mathematics methods course taken by the PSETs three semesters prior to their student teaching placement. Selected CGI elements present in the assessment were: (a) solving word problems, (b) using CGI problem types, (c) exploring solution strategies, (d) sharing solution strategies, (e) inviting multiple strategies, (f) expecting solution strategies, (g) building on student starting points, and (h) intentional listening to student thinking. I hypothesized that scores on the CGI framework assessment might be related to participants' use or non-use of CGI elements during their first lesson. The nature of the relationship would be co-occurring at best and could not be generalized or quantified with any data gathered in this study. Some of the questions included:

1. CGI helps students learn through
   a. Memorizing procedures
   b. Exploring situations
   c. Memorizing problems
   d. Practicing procedures

2. Using CGI often involves
   a. Acting out a story
   b. Memorizing answers
   c. Reflecting on ways to solve a problem
   d. Rewriting a problem to make it a story

5. Children’s solution strategies
   a. Show students’ intuitive abilities to solve problems
   b. Are often contradictory and usually incorrect
   c. Should be narrowed to one strategy
   d. Show students’ inaccurate language in describing the solution

Self-efficacy assessment. The mathematics self-efficacy assessment (known to participants as the *Pre-Assessment Informal Survey*) was given to the 10 potential candidates one time, before their student teaching placements began. The 10 open-ended questions sought PSETs' characteristics in several areas: feelings, experiences, and opinions about mathematics and mathematics instruction. The self-efficacy assessment was designed to find possible convergences and divergences between the PSETs' characteristics and PSETs' utilization of CGI elements in lesson plan creation and enacted lessons. No time limit was given and some students returned the assessment up to two weeks later.

***Self-efficacy assessment design*.** The self-efficacy assessment was designed to elicit data on PSETs' personal feelings and experiences about mathematics and mathematics teaching. The assessment was created through a qualitative lens that valued the phenomena of teaching mathematics in the natural setting (Yin, 2012) of student teaching. It was constructed with previous research that suggested PSETs' experiences may explain the relationship between content knowledge and personal teaching efficacy (Newton, Leonard, Evans, & Eastburn, 2007). The constructs assessed were: (a) feelings about mathematics, (b) how long ago they had taken a mathematics courses, (c) personal experiences, (d) mathematical highlights, (e) effective models/mentors, (f) feelings about teaching children, and (g) feelings about teaching mathematics in their upcoming student teaching placement.

The 10 self-efficacy assessment questions were:

1. How do you feel about mathematics?
2. What is your math ACT score?
3. When did you last have a math class?
4. What experiences have you had learning mathematics?

5. Was there a life moment that got you excited about math (i.e. learned how to relate math to everyday things)?
6. Describe a favorite math teacher:
7. What characteristics does a good math teacher have?
8. What are your feelings about teaching elementary mathematics?
9. What are your feelings about teaching math in your student teaching placement?
10. Should elementary mathematics be taught a certain way or with certain curricula?

Pre-LPI lesson (Lesson One). The PSETs wrote a pre-LPI mathematics lesson plan (called Lesson One) and then taught it while being observed. For this first lesson, the PSETs were asked by the researcher to use the cooperating teachers' curriculum and to include some problem solving in a large group setting. For consistency, PSETs were required to use the institutional boilerplate lesson plan format for all three lesson plans (referenced in Appendix C). The lesson was audio and video recorded, using redundant audio recording, including a lapel microphone on the participant and an area microphone to pick up conversation from the classroom. A digital recorder was used that was able to pick up SMART Board screens and LED television images. During the lesson, the researcher wrote analytic memos to record thoughts, ideas, and impressions of all aspects of the observed lesson (Miles et al., 2014).

This first observation helped establish a baseline of what PSETs planned, enacted, and modified as they taught their first lesson. Immediately following this first lesson, a post-lesson *conversation* took place in a separate, semi-private space to utilize the recency effect and to help PSETs be more accurate and honest when recalling details of the enacted lesson. This conversation was recorded and analytical memos were handwritten by the researcher during the conversation. The conversation was inductively oriented, open ended, and less structured than the formal interview, which came later in the day. This post-lesson conversation started with a general, “How did the lesson go?” (Appendix G). The conversation was intended to follow the PSETs thoughts about the lesson and their impressions of student thinking.

Later in the day, or at the end of the day, a semi-structured interview took place, again in a semi-private space. This interview was digitally recorded, and analytical memos were handwritten by the researcher. This semi-structured interview asked general questions about CGI elements that occurred, as well as any thoughts about their teaching and children’s solution or counting strategies (the interview protocol is in Appendix G).

The Lesson Plan Intervention

After the first lesson observation, conversation, and interview, the researcher and each PSET engaged in two LPI sessions to review the principles of CGI framework and to co-construct a lesson plan (called the LPI lesson) for the second enacted lesson. The two sessions reviewed the elements of CGI framework and how they can be leveraged to elicit and foster student thinking in an early number lesson. Emphasis was placed on specific CGI elements that have been demonstrated to elicit and support student thinking, as evidenced through dialog about early number sense and problem solving strategy use (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996).

Framework for the LPI. The theoretical framework of this LPI came from several sources. Taylan’s (2016) study found a positive correlation between preservice mathematics teachers’ lesson analysis skills and “their attention to and interpretation of student thinking and learning” (p. 337). Sun and van Es (2015) discovered the tendency of preservice teachers to focus better on student thinking if they had lesson analysis skills that focused on noticing student thinking. National Council of Teachers of Mathematics’

(2000) report emphasized the need to focus on student thinking when writing lesson plans, especially with preservice teachers' common practice to focus on organization and classroom management. Hughes and Smith (2004) found that using a protocol tool helped teachers consider student thinking as they wrote lesson plans. With these studies connecting lesson plans with attending to and building student mathematical thinking, the idea of a lesson plan intervention seemed a viable and efficacious route of study. Since CGI framework offers specific tools for teachers to tap into student math thinking for the purposes of building number sense, it made sense to interweave CGI elements with a lesson plan intervention.

LPI first session. The first LPI meeting lasted from 1 to 1 1/2 hours and included both direct instruction as well as discussion-based activities. This meeting began with a short discussion between the researcher and each PSET (one to one) about the general idea of the research and how it was laid out. It was anticipated that sessions with just the researcher and one PSET at a time would effectively communicate CGI material more effectively and allow for comparisons of the researcher's instruction with each PSET. This first meeting then reviewed key elements of CGI framework for the purpose of recalling earlier CGI material taught in the elementary mathematics methods course and to talk about misunderstandings or forgotten content matter. Key CGI elements included the importance of understanding student thinking and leveraging student thinking when teaching to support sense making, knowledge of CGI framework story problem types, and knowledge of student strategies for solving these problem types based on CGI research. This review included a PowerPoint presentation that included key teacher stances when using CGI practices: (a) teachers ask students how they solve problems, (b) teachers often ask student(s) if they can solve a problem another way, (c) teachers believe students have natural problem solving ideas, (d) teachers believe students' intuitive problem solving abilities are a basis for constructing meaning, (e) teachers expect ranges of solutions, (f) teachers situate themselves as intentional listeners to student math thinking, (g) teachers demonstrate willingness to struggle with identifying students' math thinking, and (h) teachers build on students' starting point(s) to progress understanding (and other components, which are listed in Appendix D). During the PowerPoint discussion, the researcher and the PSETs analyzed five or six short video segments of young students solving word problems. Videos were examined for kinds of strategies used and for levels of strategy sophistication. The researcher and PSETs examined examples of student written/drawn work. Some examples came from Carpenter et al. (2015) and included scripts of the conversations.

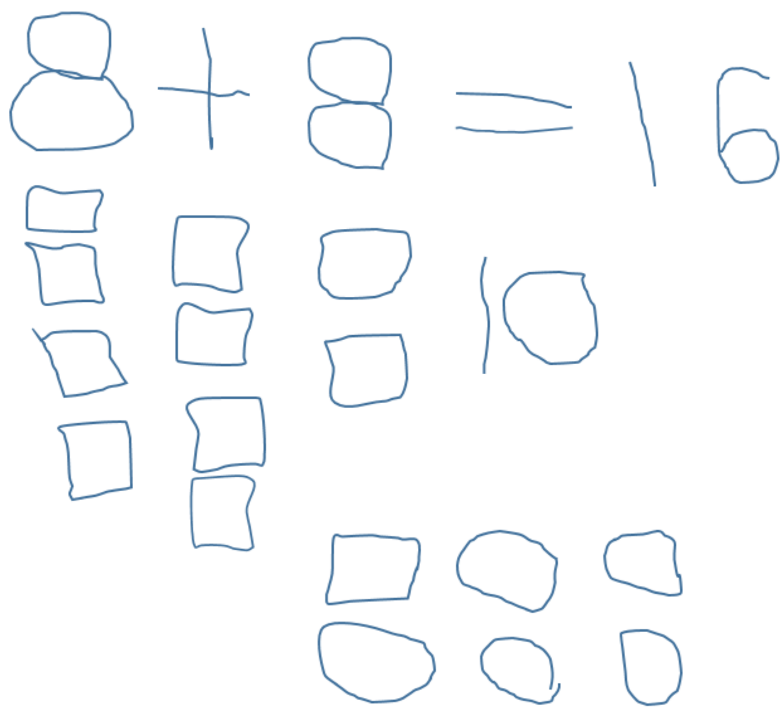


Figure 4. Example of student's solution strategy for solving 8+ 8 = ___

Included in the first LPI session were discussions about: (a) What questions might you ask that student about how she solved the problem?, (b) Why would you follow up with this student's solution within a whole class discussion?, c) What misconceptions do you foresee happening?, and (d) What do you think this child knows about this concept?

The PSETs were then asked to begin writing a second mathematics lesson plan in early number with CGI elements in mind. The PSETs could use any curriculum they wished, but were asked to include a large-group problem solving session in the lesson. They were told that the second meeting would be mostly for finishing the lesson plan with lots of time to work out how CGI elements could be integrated into it. The researcher's background in teaching with CGI practices was integral to how the PSETs would practically put CGI elements into their problem solving section of the lesson plan.

LPI second session. The main purpose of the second meeting was to co-construct the LPI lesson plan, integrating CGI elements into large-group, story problem instruction. The PSETs and the researcher worked together to accomplish the PSET's curriculum goals while integrating CGI elements into the lesson plan. The second LPI meeting took place shortly after the first and lasted from 70 minutes to 1 1/2 hours. The second session took place with the researcher and one PSET at a time. In one instance, the second session took place on the college campus in a semi-private computer lab with a table and a large screen television. The other two instances took place in classrooms at the PSETs' placement schools. This second meeting reviewed the PSET's draft of the LPI lesson plan the PSET had begun one day earlier. Two of the three PSETs had started the LPI lesson plan prior to the second LPI meeting, one (Eleanor) had not. The researcher required that at least two word/story problems be included in a large group teaching session. This was needed to put both student and PSETs' thinking out in the open and to see if CGI-based elements would help the PSETs attend to and interpret student strategies. The second session included conceptual as well as practical ideas of building CGI elements into lesson plans. The researcher asked how the PSETs were planning to leverage student thinking to build meaning and how to get students to share their thinking for themselves as well as for others' perusal. Specific teaching phrases were discussed such as, "Tell me how you solved that" and "Can you show me what you did with the counters to solve this problem?" In the second session, the three PSETs asked lots of questions about what they perceived might come up as obstacles during instruction. Other issues included what to do with student errors, how to handle misconceptions, and getting students to share more details about their strategy. One PSET asked if they should ever give a "right answer" if none were brought up. The researcher assured the PSET that accuracy is important and that the art of keeping first graders on task while asking important strategy questions was not easy, but possible. Also discussed were ways of helping students listen to each other's thinking and to be open to talking about their questions and ideas. In Chapter Four these were called "reflections", and many instances emerged of this phenomenon. The second LPI meeting also included evaluating how parts of the lesson plan format lend themselves to CGI practices while simultaneously ensuring that the lesson's goals were being met.

This co-constructed LPI lesson plan was to be used by the PSETs when she taught the next lesson. The lesson was then video recorded in their placement classroom. During the enacted lesson the researcher was also writing analytic memos.

LPI lesson plan two. This co-constructed LPI lesson plan (called Lesson Two) was used by the PSETs when they taught the second lesson of the research. The lesson was video recorded in their placement classroom. The same data collection procedures were used as in the first lesson, including video recording and post lesson conversations and interviews. Also, it was different from the first lesson interview, the researcher wanting to elicit thoughts about what effect, if any, the co-constructed LPI lesson plan had on PSETs' actions during the lesson.

LPI enacted lesson three. Two or three days after observing the LPI lesson (Lesson Two), a third early number lesson was created by the PSET and observed. This lesson plan again used the college's standard lesson plan format, using whatever curriculum the cooperating teacher required of the PSETs. The PSETs were asked to include a large group activity involving problem solving. The observation was video recorded with a lapel microphone on the participant and an area microphone to pick up conversation from the classroom. Immediately after the lesson, a post-lesson conversation took place, taking advantage of the recency effect. Later in the day or at the end of the day, a semi-structured interview (Appendix G) elicited PSETs' thinking about the lesson plan, the enacted lesson, CGI elements, and student thinking. After the interview, analytic notes were made recording impressions and thoughts about the day.

Researcher Background and Stance

Impetus for this research came from the researcher's 24 years of teaching elementary and middle school mathematics in public and private schools. My introduction to CGI framework twenty years ago changed my pedagogy to be more responsive to students' mathematical thinking. The CGI framework helped me better understand the need to intentionally elicit and support student thinking as they are engaged in early number lessons. CGI framework training helped me value the richness of student mathematics knowledge and how they use it to solve problems and share their solution strategies. Knowing that there are categories of problems which are inherently more difficult than others was not new to me, but the analysis of these categories was. CGI research validated experiences I had that there should be a way to organize strategies and classify their relative mathematical sophistications. The categories of problems and problem solving strategies children intuitively used helped me understand the level of sophistication children use in their thinking and problem solving. Sophistication, in this context, means the degree of mathematical growth that students generally go through as they grow in number sense and problem solving. Other recent research investigating children's misunderstandings of mathematical representations relates to my interest using CGI framework's emphasis of probing student thinking for error or misinformation (Cramer, Ahrendt, Monson, Wyberg, & Miller, 2017).

The researcher's stance was as an observer while recording the participants' lessons in the classroom and changed to participant-observer during the LPI meeting sessions. During classroom observations the researcher was visible, as an unnatural part of the climate of the classroom (Patton, 2015). Care was taken to work with the cooperating teacher and student teacher to ensure that students would not be unduly disturbed in the classroom, whether academically, socially, or emotionally. Overt effects were minimized through introductions of me as "Penny's teacher at college" and other such phrases. Acknowledging my presence in the classroom was important for young students, as they are sensitive to unknown adults in their classroom environment.

The researcher's stance changed to participant-observer during the LPI training sessions (Cohen et al., 2011). This was necessary so that the intervention lesson plan would adequately integrate CGI framework. The third lesson plan, like the first, was written by the PSETs without assistance from the researcher, and allowed for initiatives and thinking based solely on PSETs' desires.

Data Sources

Data came from: (a) lesson observations, (b) post lesson conversations, (c) post lesson interviews, (d) lesson plans, (e) analytic memos, (f) intervention session transcripts and (g) notes taken in other conversations. Figure 5 represents the analytic notes framework for recording notes, memos and codes of PSETs' activities during the lessons and intervention training sessions.

Time of event	Excerpt/Notes	Memos	Codes

Figure 5. Analytic notes framework. Framework includes observations, conversations, LPI meetings, interviews, and convergence of data for this intervention study.

Coding of CGI elements. Initial and axial coding of data that related to CGI concepts and elements were generated from a non-coded framework. As I went through the transcripts I tried to operationalize constructs in the context of my data. I made changes as I went, consolidating or splitting constructs into a workable set of descriptive codes for what the PSETs planned, discussed, or taught to their students. I did not consolidate a list beforehand, but regrouped and adapted some concepts according to what CGI research practitioners discussed in CGI literature. Then, when I had what I eventually called the 18 elements of CGI framework, I analyzed all the transcripts again and assigned one (or multiple) codes to all the data. The analytical process was deductive in nature, with many iterations of recoding. My focus was to capture as many CGI elements as possible in the data while trying to not manufacture false instances of elements. I hoped to collect obvious instances of CGI elements while being true to what I thought the PSETs were saying and doing. CGI framework coding helped find patterns and themes related to PSETs' use of CGI elements, student thinking, and mathematical ideas.

Table 1

Codes, designations, and descriptions from analyses of data. Some concepts taken from Carpenter, Fennema, Loef-Franke, Levi, and Empson's Children's Mathematics, Cognitively Guided Instruction, p. 14. Copyright 2015 by Heinemann.

Code	Designates	Definition and Some Examples
Invites sharing	Invites students to share solution strategies	Students are asked how they solve problems. "Would someone like to come to the board and share their strategy for this problem?"
Broadcasting	Broadcasting of student thinking	Student thinking repeated for whole group to hear. "David said that he counted from 6 to 12."
Invites multiple strategies	Multiple solution strategies sought	Students are prompted for multiple solution strategies. "Does anybody else have a different strategy they would like to share?"
Word problems	Word problems used in instruction	Students experience word problems in their mathematics instruction.
Expects strategies	Expects students to pursue solution strategies	Students are expected to pursue solution strategies.

Problem types	CGI framework problem types	Students are challenged with range of CGI framework problem types.
Intentional listening	Intentional listening to student thinking	Students experience being listened to by the PSETs.
Invites reflection	Inviting student reflections	Students are invited to reflect and respond to other students' math strategies. "Would someone like to share what you think about Jane's strategy?"
Pursues to completion	Pursuit of student thinking to completion	Students experience dialogue that continues to problem completion.
Correlates	Problem types correlated with solution strategies	Students experience correlation of strategies with appropriate problem types.
Starting points	Building on student starting points	Students experience teacher's use of their initial ideas.
Invented	Invented algorithms used by students	Students show use of invented algorithms - three types possible.
Presenting problem	PSETs present word problems without modeling strategies	Students are presented word problems without PSET's modeling of any solution strategies.
Respect	Respecting others' thinking	Students experience respect for their math thinking in the classroom. "Thank you, Jose, for sharing your strategy. Why did you start with . . . ?"
Intuition	Children as intuitive problem solvers	Students experience teacher's using their intuitive strategies.
Teacher learning	Teachers learn from listening to children	Teachers learn about children's thinking while listening to them solve problems.
Flexible range	Flexible number sets used in instruction	Students can choose number sets they are comfortable with.
Teacher struggle	Teachers show struggle when grasping thinking	Teacher willing to struggle to understand student thinking.

Data Analysis

Analyses began when each PSET's first lesson was observed (Patton, 2015) through the use of analytic memos. Miles et al. (2014) describe an analytic memo as "a brief or extended narrative that documents the researcher's reflections and thinking processes about the data" (p. 95). Analytic memos served to integrate and cross-check data for initial instances of CGI elements, PSETs' teaching actions, occurrences of student thinking, counting strategies, extending counting strategies, and commenting on other students' comments.

The researcher looked for patterns of instances to warrant any assertion(s), or sets of assertions, that led to themes and categories. Analytic memos also assisted with reflections about similarities and differences in PSETs' actions, discourses, student thinking, and related events in real time. Using elements of CGI framework as units of analysis, an example would be the CGI element of "teachers demonstrate willingness to struggle with identifying students' math thinking". When this element was observed, it was an instance of a CGI element and counted as a pattern if found in other places as well. Text from a lesson plan would be coded as a CGI element if it appeared to encapsulate one of the 18 elements. Sometimes lesson plan data would fit two or three elements, and was coded as such. Data from an observation was coded as an element if the PSET spoke some of the language of the element as she was teaching or if the PSET enacted a common form of the element as described in CGI literature. Data from conversations and interviews with PSETs was coded as a CGI element if CGI language was used or commonly accepted synonyms for CGI language was observed. Data from interviews was also coded as a CGI element if the PSET generally described the main idea of an element. Cycles of data coding and analyses allowed for inferences and patterns to develop and mature.

*Figure 6***.** Analysis and convergence of data for this intervention study. Convergence idea is from information in Miles, Huberman, and Saldana (2014).

A sincere effort to take an ethical stance to data analysis was followed by the researcher (Merriam, 2009). A constant comparative approach to data analysis was used to make data analysis a process oriented activity (Cohen et al., 2011; Glaser & Strauss, 2017; Miles et al., 2014). There were successions of deductive coding, memoing, and theme making to look for patterns or unique data. Analytic memos were taken to record thoughts and perceptions about PSETs' use of CGI to elicit and support student thinking. Immediately after the lesson, a post-lesson conversation took place to gather recent data. The researcher analyzed data with care so that enough instances of an occurrence would be present before assertions could realistically be warranted. With three participants in the study, generalizations would not be warranted, but conclusions are sound under the circumstances in this study.

As patterns and themes emerged, differences within and between cases were compared, making sure that assertions were warranted. Cross case analyses were conducted; the goal was to examine emergent data from each PSET's lesson planning and teaching to discover patterns or themes related to the LPI, CGI, and children's mathematical thinking. Care was taken to look for disconfirming evidence and to intentionally focus on data more than interpretations.

Instructional definitions for question three. To help answer question three, descriptions of teaching strategies are necessary. Direct instruction is characterized by teacher-directed dialogue with occasional student interaction. Limited inquiry is posing a general initial question with limited follow up questioning. Full inquiry is characterized by Socratic practices, such as asking questions, general and specific follow up questions, and pursuing student-generated thinking. Large group mini-lecture is characterized by a combination of lecture, modeling, and one to one discussions.

Trustworthiness

Trustworthiness addresses possible biases and analytical misinterpretations of the researcher (Miles et al., 2014). The data collected could be influenced by the author's bias in the belief of CGI framework validity as a useful model for not only mathematics instruction but also as an effective framework for uncovering, supporting, and extending student mathematical thinking. The advantage to this is that the author had knowledge of the PSETs' background in CGI research in student thinking. All of the participants had taken (and passed) the author's elementary mathematics methods course 15 months prior. The language and thinking of the author are known to the (potential) participants and this could reduce the misunderstanding of terms and protocols of CGI framework for teaching. Lincoln and Guba's (1985) research supports the advantages that time with a researcher can have on participants' willingness to trust the researcher.

Care was taken to clarify for participants that their lesson planning, teaching, and interviews were not part of their grades or edTPA scores. They were also notified, in writing, that their responses, actions, inactions, and communications would affect only the content of the study and that member checking (respondent validation) was a welcome part of the research process (Miles et al., 2014).

Biases stemming from the author's effects on the PSETs was partially mitigated by the participants' familiarity with the researcher (Miles et al., 2014). The participants knew that the researcher has a growth stance, meaning that a critical look at PSETs' teaching and planning is always meant for learning purposes and not evaluating purposes. A year previously, one student (not in this study) agreed to a pilot study of mathematics conversations in a public school elementary classroom. The nuances and complexities of research methodology helped the author to understand the realities of PSETs' expectations and pressures while teaching in a placement classroom.

Another point of vulnerability could have come from participants' possible feelings of having to participate in the study, for various reasons. One reason could be the perception that participation was not really voluntary, or that their agreeing to the study was a statement of the validity of CGI practices as a realistic or valuable model for a lesson plan intervention. It is also possible that the PSETs could feel that their knowledge of CGI framework and its tenets were not satisfactory for the researcher's needs or expectations (Miles et al., 2014). An advantage of this methodology was that the PSETs were not under the supervision of the researcher for their student teaching requirements, responsibilities, and supervision. Different expectations and assumptions could have been made by both the researcher and the PSETs had this not been the case. Miles et al. (2014) state the need to have checks for effects of the researcher on the case and effects of the case on the researcher. It is unrealistic to think the PSETs did not have expectations for how much the research would affect their student teaching responsibilities as far as time constraints and added stresses. Frequent checks with the PSETs were given to ensure they felt they could continue the research as the 6 to 10 day stretch progressed. The researcher's expectations were that the PSETs would welcome the opportunity to be involved in research while simultaneously acknowledging the extra effort and time it would take to be a participant. The researcher also recognized that his desire was to see the LPI be successful, or at least partially successful, in leveraging CGI practices to elicit student thinking. The training sessions were intense content-wise, and it affected my expectations that at least some of the thinking would manifest itself in their second and third lesson plans and enacted lessons. My expectations were also positive that the PSETs would think conceptually about the mathematics they were going to teach, and would recognize this need students have as they endeavor to make sense of the mathematics in the word problems.

Internal validity in qualitative research is low (Patton, 2015), especially with low numbers of cases/units under analysis. However, some validity was possible through the correlation of the five data sources: (a) observations, (b) conversations, (c) interviews, (d) analytic memos, and (e) lesson plans. Triangulation from different data sources (LPI event, lesson plans, video recorded lessons, interviews, and analytical memos) and from different methods (recordings, interviews, and evaluations of lesson plans) helped ensure a proper weight of evidence was present before an assertion could be made (Denzin & Lincoln, 2000). Frequent, intentional, and systematic analysis of data was present in second cycle coding. Key terms and elements in CGI framework were cross-checked for presence in the lesson plans, lesson planning sessions, enacted lessons, and interviews. Cross checking of PSETs' observations with PSETs' interviews reduced unwarranted assertions about why the PSETs supported or didn't support student thinking during the enacted lesson. Cross checking of the lesson plans with the enacted lessons (including the LPI lesson) clarified what elements of CGI framework the PSETs believed was important for eliciting and supporting student thinking to build number sense.

Assumptions of the researcher included PSETs knowing the tenets of CGI framework from their elementary mathematics methods course. PSETs also had a beginning knowledge of student thinking in general, and of early number mathematical thinking in particular, from previous experiences with children in two separate practicum placements (about 150 hours). Some of the PSETs had a greater mathematical knowledge for teaching (MKT) and pedagogical content knowledge (PCK), based on the researcher's experiences with the PSETs in his elementary mathematics methods course. Although no pre- or post-test mathematics content assessments were given, the two self-assessment surveys gave a general idea of each PSET's comfort level with content and teaching. Additionally, ACT scores, methods class conversations, post-lesson conversations, and interviews all contributed to a general idea of each PSET's mathematics knowledge, confidence, and teaching comfort.

Limitations for this research included a small n (3), limiting the generalizability of any relationships about the LPI's effect on PSETs' integration of CGI elements in lesson plans and their consequent enactment/non-enactment during instruction. Limitations also included the number of lessons observed, set at three for each PSET studied. A more robust model would be required for a stronger generalizability of the results that emerged, i.e. greater number of PSETs with similar learning environments.

The scope of the proposal was limited by the narrow look at supporting student thinking through the model of CGI framework. Other models and theories which support student thinking are equally as valid and effective for leveraging student thinking to grow mathematical knowledge (Ball, 1993, 2007; Ball & Bass, 2003; Gess-Newsome, 1999; Thompson, Carlson, & Silverman, 2007). Other limitations also included the realities of the number and differences of placement schools available, the willingness of the PSETs to participate, and the relationships they felt with the research and the researcher. As the researcher was not in an evaluator role over the PSETs, the results can be said to be reliable under similar circumstances (non-supervisory, small *n*, fairly similar classroom demographics, and similar exposure to CGI elements and practices).

From Purpose to Enacted Research

A short pause here to give the reader a brief summary of this study. The purpose of the study was to help student teachers elicit, interpret, and utilize student mathematical thinking through a CGI framework-based LPI. Chapter One presented the rationale for the research and described the three research questions that arose from there. Chapter Two described the field of literature that addressed the touchpoints of this study - student teachers, mathematics, lesson plans, CGI framework, and interventions. In Chapter Three the methodology of the study was laid out: the research design, structure, data collection, data analysis, LPI process, and elements of CGI framework including mathematical problem types and solution strategies. Chapter Four describes the LPI process as it was enacted and examines the PSETs' lesson plans, enacted lessons and student mathematical thinking.

Chapter Four: Findings

The purpose of this study was to see if a CGI framework-based LPI would bridge a three semester time gap between PSETs' mathematics methods course and their student teaching course. The bridge was a lesson plan intervention (LPI), conducted in the spring of 2018 to help PSETs integrate elements of CGI framework into their lesson plans to elicit and utilize student mathematical thinking for problem solving. Observations of these enacted lesson plans were then analyzed for the presence of CGI elements (Carpenter et al., 2015).

As a reminder, the three research questions were: (1) "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?", (2) "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?", and (3) What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?"

Chapter Four Overview

Chapter Four articulates the findings of the study, focusing on the presence and utilization of CGI elements by PSETs to elicit and utilize student thinking. Also included are tables listing the presence of CGI elements in PSETs' lesson plans and enacted lessons. The chapter then describes and analyzes each PSET's utilization of four CGI elements. The first level of analysis was by individual PSET, forming three cases to this study. The second level of analysis was by CGI element, looking at how they were utilized by each PSET. The third level looked for changes in PSETs' use of the elements over time.

Of the 18 identified CGI elements I looked for in the PSETs' lesson plans and enactments, six were prominently used by the PSETs. These six were chosen because of their significant use by at least one PSET and because they were an important part of how the PSETs elicited and leveraged student thinking to build number sense. Three of the six elements were common to all three PSETs' work and three were unique to each PSET (one each). The three common elements were *broadcasting, invites students to reflect,* and *invites multiple strategies*. The three remaining elements, *builds on student starting points, willing to struggle to understand student thinking,* and *pursues student thinking to completion*, were chosen individually for analysis because of their unique incorporation by a PSET. This combination of three common elements and one unique element meant that each PSET's lesson planning and enacted teaching would be analyzed through four CGI elements. This design shed light on the possible effects of the LPI sessions on PSETs' utilization of these elements over two subsequent lesson plans and enacted lessons.

A reflection on the LPI sessions revealed possible reasons why these six elements were present at various points in the PSETs' practices. In each of the first LPI sessions, I reviewed (via PowerPoint and CGI framework video vignettes) the 18 CGI elements. Two of the six elements (*broadcasting* and *pursues student thinking to completion*) occurred in several of the videos. The instances occurred as instructors guided the children through word problem solutions. It seemed probable that the first LPI sessions had some influence on the PSETs' decisions to integrate these two elements into their planning and instructional practices.

The PSETs' integration of another of the six elements, *invites multiple strategies*, into their lesson plans and enacted lessons was probably not just a co-occurrence with the element's presence in the LPI sessions, as this element was enacted in only one of the PSETs' first lessons. In the sessions I often modeled this element with phrases such as, "That's the value of this [CGI instruction], that they begin to learn from each other the different ways of solving a problem."

Also reviewed in the LPI sessions was the element *willing to struggle to understand student thinking*. With the PSETs I shared versions of:

> They demonstrate a willingness to struggle with identifying their [students'] thinking. Taking time to really listen, "I think this is what you're saying, you tell me is this what you're saying, or are you saying something different?" So it's very student-centered.

Additionally, in the first LPI sessions I taught the PSETs about the CGI element *invites students to reflect on other students' thinking*. In the second LPI sessions I encouraged the PSETs to incorporate this element by asking their first graders questions such as, "What do you think about what Susie did?" The last of the six common elements, *builds on students' starting points*, was likewise reviewed in the first LPI training sessions. In the second LPI session this element was integrated into their second lesson plans through asking general follow up questions, with the PSETs being encouraged to *listen intentionally* to student thinking.

Research Questions One and Two

Research Question One asked, "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI (lesson plan intervention)?" Research Question Two asked, "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" Each case will be presented in detail in the following case headings. The above two research questions are answered at the end of each case.

Jennie as a Case

Jennie was situated as a unique participant for this study. She was completing her bachelor's degree in education along with licensure programs in elementary education and an additional licensure in pre-primary education (ages 3 years to third grade). Jennie was the only participant so situated - the other two participants sought middle school licensure or no pre-primary licensure. In my mathematics methods course Jennie participated actively and was sometimes confused about some of the CGI elements and their purpose in mathematics instruction. Her initial responses to some of the discourse practices in CGI framework were positive and she presented herself as curious about how problem types, solution strategies, and discourse practices could be practically woven together to build students' number sense.

Before the study began, Jennie was given two pre-assessment informal surveys to form a baseline for her mathematical background, experiences, and feelings about teaching mathematics in her placement classroom. Her first survey revealed that Jennie's recollection of CGI elements from the previous mathematics methods course was somewhat fragmented. Jennie remembered some main ideas like CGI framework problem types and corresponding solution strategies, but not as much about how to elicit student thinking and leverage it for sense making. When asked how she felt about mathematics, Jennie replied,

> Not very good. I love it and it is super fun when it clicks but that only happened on rare occasions and in certain concepts. I think a part of this may be caused by my math instruction growing up, since my mom also struggled with math growing up and may not have known multiple ways to explain a concept. I've always had a hard time with math and remember one time in elementary school I did so poorly on a math worksheet that I had to redo it and ended up being sent to my room to stop crying since I was so upset about it.

In the self-efficacy pre-assessment Jennie expressed another concern about teaching mathematics - that she would underserve her students due to her lack of math knowledge. "I'm worried about attempting to explain math concepts. While I can follow a procedure I don't always know the reasoning behind it." Jennie's struggles with mathematics during past stressful experiences are common to many PSETs. In light of these struggles, it was impressive that Jennie was willing to participate in this study and did so with a sincere desire to experience how CGI framework thinking could be practically utilized.

The following sections highlight Jennie's integration of four CGI framework elements into her lesson plans, LPI sessions, and enacted lessons as they occurred in an authentic learning environment. The four elements were: (a) *broadcasting*, (b) *building on students' starting points*, (c) *invites students to reflect on other students' thinking*, and (d) *invites multiple strategies*. Through Jennie's use of these four elements, data emerged that helped me understand how she utilized CGI practices as a tool and framework to help her elicit, interpret, and utilize student thinking to develop her first graders' number sense.

Jennie's utilization of CGI elements.

Table 2

*Elements of CGI framework present in **Jennie's Lesson Plans**. Each row is the same CGI element. There were 18 CGI elements available for Jennie's use, only some of which she integrated in each lesson plan. The four highlighted elements were central to Jennie's instruction and were analyzed as Jennie integrated them into her three lesson plans.*

Pre-intervention Lesson Plan (1)	Intervention Lesson Plan (2)	**Post-**intervention Lesson Plan (3)
Uses CGI framework problem types	Uses CGI framework problem types	Uses CGI framework problem types
Uses word problems to build number sense	Uses word problems to build number sense	Uses word problems to build number sense
Invites multiple strategies	Invites multiple strategies	Invites multiple strategies
	Builds on student starting points	Builds on student starting points
Expects students to pursue solution strategies	Expects students to pursue solution strategies	Expects students to pursue solution strategies
Invites students to share solution strategies	Invites students to share solution strategies	Invites students to share solution strategies
Broadcasts students' thinking to class	Broadcasts students' thinking to class	Broadcasts students' thinking to class
6 elements ↑	Invites students to reflect on other students' thinking	Invites students to reflect on other students' thinking
	Intentional listening to students	Intentional listening to students
	Flexible range of numbers	
	10 elements ↑	Teacher learns after listening to students
		Presents math problems without modeling any
		11 elements ↑

Table 3

Elements of CGI framework *in* ***Jennie's Enacted Lessons***. *Each row is the same element. The four highlighted elements were central to Jennie's instruction and were analyzed as Jennie enacted them in her lessons before, during, and after the LPI.*

First Lesson	Second Lesson	Third Lesson
Expects students to pursue solution strategies	Expects students to pursue solution strategies	Expects students to pursue solution strategies
Invites students to share solution strategies	Invites students to share solution strategies	Invites students to share solution strategies
Broadcasting	Broadcasting	Broadcasting
Presents problems without modeling	Presents problems without modeling	Presents problems without modeling
Builds on students' starting points	Builds on students' starting points	Builds on students' starting points
Invites multiple strategies	Invites multiple strategies	Invites multiple strategies
Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities
	Pursues student thinking to completion	
7 Elements ↑	Correlates problem types with solution strategies	
	Intentional listening to student thinking	Intentional listening to student thinking
	Invites students to reflect on other student's thinking	Invites students to reflect on other student's thinking
	Uses CGI framework problem types	Uses CGI framework problem types
	Uses word problems to build number sense	Uses word problems to build number sense
	13 Elements ↑	11 Elements ↑

Jennie's use of broadcasting. *Broadcasting* is my descriptor for the teaching practice of making a student's statements, questions, or reflections known to the whole group. In CGI framework literature, broadcasting is intended to illuminate students' thinking and solution strategies for the class to observe. It gives other students a chance to hear or see the mathematical thinking of another student so that multiple strategies become noticeable for cognition. *Broadcasting* is an important part of orchestrating mathematical discourses (Carpenter et al., 2015). Jennie incorporated broadcasting in all three lesson plans and all three enacted lessons. We discussed broadcasting during the LPI sessions. Looking at *broadcasting* through Jennie's participation in the LPI brought out important aspects of how she utilized broadcasting to elicit student thinking.

Broadcasting in lesson plan one and enacted lesson one. In Lesson Plan One (before the LPI) Jennie included two objectives, "I can take apart word problems in order to solve for the missing partners" and "I can solve missing partner problems". Her two planned problems were, "There are (12, 14) puppies, some are brown and some are black. How many are brown and how many are black?" Twelve of the 18 students volunteered to share their thinking in the lesson. In this lesson plan I looked for *broadcasting* and found it in her statement, "Students will also be given the opportunity to be the teacher by coming to the board to show their thinking." Jennie's use of broadcasting had both a visual and oral representation at this point in the study. She also incorporated broadcasting by having them write their equations on the board. Equations, as written symbols, are another type of representation of a concept (Lesh et al., 1987). The presence of these three types of broadcasting gave evidence to Jennie's desire to elicit student thinking in multiple ways.

In Enacted Lesson One, Jennie demonstrated *broadcasting* many times. Some of these instances occurred when students shared their equations on the SMART Board and others occurred when they explained their thinking orally and Jennie repeated their statements. In one of these discourses Jennie spoke with a student, Aleson:

1 Jennie:	All right, let's see, Aleson, can you share what you did?
2 Aleson:	I think 8 + 6 equals 14.
3 Jennie:	So Aleson did 1, 2, 3, 4, 5, 6, 7, 8 [writing circles on board] plus 6. 1, 2, 3, 4, 5, 6 [Jennie drawing 6 circles on board] Does that equal 14? We have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14.
4 Aleson:	I know it because 6 + 7 is 13. And one more is 14.
5 Jennie:	Hmmmmmm. Interesting thinking going on.

Jennie's first sharing about Aleson's response (talk turn 3) was an assumption about Aleson's strategy. Though Jennie jumped in (talk turn 3) before Aleson finished her thinking, Jennie worked hard to illuminate one solution strategy for student contemplation. Since this observation took place in the first lesson, it served as a baseline for what Jennie perceived and practiced as broadcasting. Jennie seemed to value sense making in her instruction and used broadcasting of the mathematics model "math fact triangles" and equations to help students "dissect" the problem before they solved it. Her successful implementation of math fact triangles (a teaching model) and equations (a symbolic representation/translation) into her first lesson pedagogy was consistent with her lesson plan objectives.

Broadcasting in LPI sessions. In the first LPI session, Jennie and I reviewed the concept of broadcasting and its importance in giving students many opportunities to hear other students' mathematical thinking. I did not use the term "broadcasting" during the session but described the concept while watching videos of children solving problems. As

Jennie repeated a child's statement or question, I pointed out that this action was an important aspect of a mathematical discourse.

More discussion of broadcasting occurred in the second LPI session. While discussing what it meant to invite students to share their thinking, the conversation went as follows:

1 Jennie:	Maybe I will have them share from their spot.
2 Researcher:	Then you're clarifying what they do, that's totally fine. Or they can stand and share.
3 Jennie:	[writing] Could stand and share
4 Researcher:	You can have them stand and share or you can have them come up to the board, depends on what you want.
5 Jennie:	So I will have them stand and share, and I will clarify their thinking again?
6 Researcher:	I think reveal their thinking is probably a better term for that. You're just trying to make sure that what the student did, what their solution strategy is, is evident to everybody else.
7 Jennie:	Um, for the whole class?
8 Researcher:	Yes. I don't know if clarify is the best, I think "reveal" is better.

At this point it appeared the LPI was beginning to help Jennie understand the CGI framework idea of *broadcasting student's thinking* from a practical standpoint, that broadcasting should illuminate student strategies and thinking for as many students' consumption as possible, i.e. talk turn 7 "the whole class?"

As the second session of our LPI progressed, I again reinforced the idea of broadcasting to Jennie:

1 Researcher:	And I would say after this, you're going to help them clarify their thinking for the other students. You're trying to reveal what their thinking is. You are trying to make sure other students think, at least hear how they . . .
2 Jennie:	Help clarify thinking for the whole class.

3 Researcher: To clarify their solution strategy is really what you are trying to look at for the whole class. You're trying to make sure, did they do this, what did they do? Did they count on? Did they count from? Did they match? Did they pair? You're just making sure the rest of the class has an idea of what that student did. Maybe it's really clear when they shared a comment and maybe it isn't.

An important aspect of broadcasting was highlighted in this vignette - that Jennie should try to clarify students' thinking while simultaneously noticing mathematical solution strategies. This is not an easy task, even for seasoned educators. This deepened the role of broadcasting, intertwining it with a more interpretative function.

Broadcasting in lesson plan two and enacted lesson two. Jennie's integration of *broadcasting* into her second lesson plan came from the LPI sessions, mostly from the second one. In the second LPI session, we co-created her second lesson plan to integrate CGI framework elements that fit into her lesson objectives and that would help Jennie elicit student thinking. The two lesson objectives for Jennie's second lesson plan came directly from CGI framework - that students would write and solve join, result-unknown and join, change-unknown story problems. The scripted dialog in her plan included Jennie presenting one of these problem types to the class, then inviting them to solve the problem on their personal whiteboards, then giving them time to solve the problem, and finally to have them share their strategies with the class. After they shared, Jennie would ask, "What do you think about how Susie solved this story problem?" The lesson plan then asked if someone had a different way of solving the problem. Her lesson plan included two rounds of this process. In her second lesson plan Jennie wrote, "Teacher will help clarify that student's thinking for the whole class", and "The teacher will reveal the child's thinking for the whole class". Here, Jennie had adopted this added aspect of broadcasting - not merely repeating a child's statement or question, but trying to expose more of the "sharer's" thinking at the same time. From here we look at whether this deeper definition of broadcasting moved into Jennie's second enacted lesson. Nine of the 18 students volunteered to share their thinking in the lesson.

Observing her second lesson, Jennie integrated broadcasting through a short discourse with Patty:

1 Jennie:	Patty is going to share how she got her answer. What answer do you think that is?
2 Patty:	[Standing at the SMART Board] 5.
3 Jennie:	Patty thinks it is 5. How did you figure that out, Patty?
4 Patty:	Yeah, because I started with 8 and I got to 13 by counting on.
5 Jennie:	Did you guys hear what she said there? Patty said she counted on with her fingers.
6 Patty:	With dots.
7 Jennie:	Oh, she counted on with dots. So she used her white board to count on with dots. Does anybody have any comments on what Patty just did?

What was not visible from this conversation was how Jennie stopped speaking and turned to the class during her "Did you guys hear what she said there," question. Her broadcasting was intentional and although she did not repeat Patty's "started with 8" solution, she followed through with what Patty did with the dots on her worksheet. This was a different form of *broadcasting* than her first enacted lesson. In this vignette from her second enacted lesson, Jennie seemed less interested in just repeating a student's statement but was trying to interpret the student's thinking as well.

Broadcasting in lesson plan three and enacted lesson three. After teaching her second lesson, Jennie created a third lesson plan, with instructions from me to integrate story problems for large group instruction. Jennie's third lesson plan objectives asked students to “Create my own separate-result-unknown story problem” and “Find and make a 10 in story problems.” Seven of the 18 students volunteered to share their thinking in the lesson.

Broadcasting showed itself similarly in Jennie’s second and third lesson plans, with the exception that in her third lesson plan she broadcasted a specific strategy (making a 10). She wrote, “Teacher will ask students to show their thinking for solving the problem, asking them if they see any number that could be added together to ‘make a 10.’” She included this sentence three times in the lesson plan. Like her first lesson plan, Jennie’s third lesson plan broadcasted a specific strategy, although not the same one. Her first lesson plan broadcasted equations as solution strategies, while her third lesson plan broadcasted her “making a 10” strategy. In her enacted third lesson, Jennie utilized broadcasting for different purposes. Jennie invited Coopen to share his thinking:

1 Coopen:	I counted these with my fingers [child pointing to his 8 circles, 5 circles, and 2 circles and is interrupted by Jennie].
2 Jennie:	Hmm . . . Does anyone have a comment on what Coopen just said? Madeli?
3 Madeli:	How did he get that?
4 Jennie:	So how did you figure it out, Coopen?
5 Coopen:	I counted with my fingers.
6 Jennie:	He counted with his fingers. Did you count on or did you count each one of those?
7 Coopen:	I counted each one of them.
8 Jennie:	He counted each one of them. Thanks for showing us your thinking, Coopen.

In this vignette, Jennie utilized broadcasting in three ways - first to repeat a different student’s question, second to repeat a student’s strategy, “I counted on my fingers”, and third to further unpack a student’s counting strategy. The two strategies she mentioned - counting on and counting individual items, vary in sophistication level, and showed Jennie's recognition that Coopen could have chosen the simpler or more complex strategy. This demonstrated growth for Jennie, as she began to utilize the broadcasting element to make sure the whole group could hear Coopen's thinking. She also used broadcasting to ask a specific mathematical follow up question. This was a change from her use of broadcasting in her second lesson as well as her first.

Changes in Jennie's broadcasting. Jennie appeared to increasingly value *broadcasting,* as it helped her interpret student thinking. Jennie also appeared to increasingly value *broadcasting* of student comments of other students' thinking *(inviting students to reflect*). Additionally, she mentioned broadcasting as an opportunity for students to rephrase their original responses. At the conclusion of the study, Jennie shared her thoughts of *broadcasting*.

> I will say to the rest of the class, [with] the student volunteer at the front, "Does anybody have a comment on what the student volunteer said? Lisa, how did you get that?" So I'll say, "They're wondering how you got that?" And I'll be trying to think [about] what they might be thinking, when they say, "How did you get that?" Because I want my student volunteer to respond a little differently than what they first said, so I will listen to what the student volunteer says, ask for that comment, [then] the student will comment, and so they're asking, "How did you get that?" It shows how did you use your drawing, or whatever they didn't share as much about before.

Jennie's use of invites multiple strategies. My descriptor *invites multiple strategies* characterizes a teacher's act of inviting students to share a different solution strategy than one shared previously. It lets students know they are welcome to share their own way of looking at a problem and that there are many ways to solve mathematics problems. *Invites multiple strategies* is enhanced by *broadcasting* to reveal multiple mathematical concepts for students to contemplate as they build number sense.

Invites multiple strategies in lesson plan one and enacted lesson one. In her first lesson plan, Jennie demonstrated *invites multiple strategies* through her notation, "Teacher will emphasize that there is more than one way to get to the number 12." She also wrote the question, "Is there more than one answer?" on her plan. Before the LPI, Jennie showed she valued multiple strategies when solving problems.

In her first enacted lesson, Jennie told the class, "I love [that] there are so many different ideas we could do . . . I love that in math we can do a lot of different things. I'm going to be looking for a lot of different examples, there's lots of different ways to do it [sums to 12]." She mentioned *equations* 17 times in the context of looking for equations to get to 12 and that there were many ways to do this. She did not directly say, "Did anyone do this another way", but made it clear that there were multiple ways to get answers to the mathematics problems she presented.

Invites multiple strategies in the LPI. This element was present in both sessions of the LPI. The first LPI session touched on multiple strategies when I mentioned to Jennie that "Teachers ask students if they can solve problems in other ways". In the first session one of the PowerPoint slides modeled *invites multiple strategies* in phrases like, "Can you show me another way to solve this?" or "Is there another way this can be solved?" Jennie and I discussed how she could speak to students to elicit their thinking for others to hear. "I'm going to maybe ask you to solve this two ways; can you show me two different ways to come up with 17?" was an example of what Jennie could also say to students. The rest of the session had several discussions about Jennie's ideas of multiple strategies for solving problems. In these discussions Jennie primarily referred to multiple strategies as models like "math mountains, equations, or math fact triangles."

In the second LPI session, we discussed how she could invite students to use many different solution strategies and what the practicalities would look like. As we co-created the lesson plan I made it clear that *invites students* is important and that part of this invitation should be to ask for different ways of solving word problems. It sounded like this:

1 Researcher: In CGI framework you want multiple solution strategies, so have another student share how they solved this problem. Ask them again, does somebody else have a different way?
2 Jennie: [typing] Does somebody have a different way?
3 Researcher: Does someone want to share a different way of doing this? Something like this. Again we are looking for multiple strategies.
4 Jennie: Uh huh ok. [typing] Someone share a different way of doing this.

Here I gave Jennie direct instruction about implementing the element of *invites multiple strategies* into her lesson plan. As we planned together she was eager to get this element into her lesson plan and was meticulous about recording the words she would use as she spoke with the students. She mentioned different forms of mathematical representations (math mountains) and different mathematical strategies (counting on to, counting on from) as ways of *invites multiple strategies*. Jennie asked:

1 Jennie: Should I remind them like "Ok we can do the equation method, math mountain," or should I remind them of their options, or should I just say "How should we solve this problem?"
2 Researcher: We will be conscious of looking for those [tools] but we will just go ahead and let them solve the problem and have students share their solutions.
3 Jennie: What if no students have solutions?
4 Researcher: Oh, they will. They will surprise you.

Jennie was figuring out how to start problem solving sessions while wondering what capacities they had as first graders. She also wanted help navigating a discussion using the *invites multiple strategies* element. She wanted to stretch her students and yet did not want to "confuse" them. She was also beginning to wrestle with visual models, like math mountains, and whether these kinds of models should be suggested or encouraged. This vignette was also a precursor to a conversation about visual models versus quantitative strategies. Later in this section on Jennie, I discuss how the LPI sessions helped clarify for Jennie how to interpret and discuss the mathematics of a problem and not rely on a set of visual models that may or may not emulate the actions or conditions of a mathematics story problem.

Invites multiple strategies in lesson plan two and enacted lesson two. In her second lesson plan, Jennie first utilized the element *invites multiple strategies* through her two learning objectives: (a) I can write and solve a join, result unknown story problem in multiple ways, and (b) I can write and solve a join, change unknown story problem in multiple ways. Jennie's lesson plan included the phrases "Does anyone want to share a different way of doing this?" and "Did someone solve this problem a different way; could you stand and share?" This was a more direct way of *invites multiple strategies* than in her first lesson plan and first enacted lesson and showed Jennie's increased capacity for seeking multiple solutions as a way to elicit and utilize student thinking to build number sense.

Invites multiple strategies emerged from her third lesson when she asked her students different versions of "Does anyone have a different way?" It was often followed up with the phrase "How did you figure this out?" In the following vignette, Jennie invited the class for a different strategy:

> Does anyone have a different way they solved question number 2? So she used counting on. All right I like how everybody is using . . . different ideas. I'm seeing

some equation methods, and seeing some math mountains, some math fact triangles. Does anyone have a different way they solved question number 2?

Differences existed between Jennie's first and second enacted lessons in Jennie's utilization of the *invites multiple strategies* element. In her first enacted lesson, Jennie connected this element to visual/symbolic models like "math mountains, equations or math fact triangles."

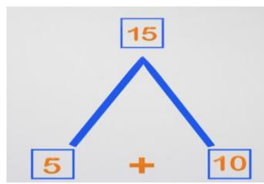


Figure 7. Jennie's visual representation of two addends and a sum - what she described as a "math mountain".

In her second enacted lesson, Jennie pursued these models, but also sought out students with "different" strategies. This change, although small, demonstrated Jennie's implementation of *invites multiple strategies* through CGI framework language such as "counting on to" and "counting on from". In one instance Jennie asked a child to show the class a way to solve a join, result unknown problem 8 + 5 = ____ :

1 Jennie: She is going to show us her whiteboard.
2 Child: I counted on from 8 and I added 5 more, I counted it and it had 13.
3 Jennie: So she used counting on. All right I like how everybody is using different ideas. I'm seeing some equation methods, and seeing some math mountains, some math fact triangles.

We see how Jennie used a modeling approach to the problem but also mentioned the strategy of "counting on", an appropriate and useful strategy for solving this problem type.

Invites multiple strategies in lesson plan three and lesson three. Jennie continued to elicit multiple strategies by asking students to share their thinking through phrases such as, “Share with us your thinking”, and “I want Coopen to share his thinking”. When a student shared an invented incremental strategy for 7 + 3 + 4 [making a 10 and adding 4] she stated:

> Wow, did you guys hear what Pam just said? Did you hear what Pam just said? She said 7 + 3 equals 10. She went 7 + 3 equals 10 [Jenny writing 7 + 3 equals 10 horizontal on the SMART Board] and then she knew there was a little 4 here we don't want to forget that [Jenny circling the 4 on the board] so she went 10 and then she knew 10 + 4 equals 14 [Jenny writing 10 + 4 equals 14 on the SMART Board] Wow, do you know how many good equations and many different ways to solve everything right now? I see a lot of crossing off, I see equations, I see math mountains; we are having a rockin’ Tuesday, that's for sure.

Jennie’s demonstrated enthusiasm for broadcasting this student’s solution strategy merged with telling students that there are many ways to solve this problem. While eliciting solution strategies from her students, Jennie continued her practice of sometimes speaking for students but also revealed her penchant for students to be exposed to multiple solution strategies. She did not use phrases like “different ways” or “another way” but did give many invitations for students to share what they thought about the problem.

Jennie’s third lesson did not reveal large changes from her second enacted lesson in her use of the *invites multiple strategies* element, but she did use it more than in her first or second enacted lessons. For this element, the LPI sessions reinforced Jennie’s practice of *invites multiple strategies*, but mostly through her favored symbolic form of using and creating equations to solve a problem. It could be said that the LPI had a small influence on Jennie's use of symbolic forms of mathematics strategies to elicit student thinking for building number sense.

Changes in Jennie's use of invites multiple strategies. Her use of *invites multiple strategies* increased after the LPI sessions and increased yet slightly more by her third lesson plan and enactment. Though Jennie did not end up asking students to compare different strategies, she enjoyed "just getting to know students' thinking, they have so much more going on in their heads than I think we give them credit for".

Jennie and invites reflection. The descriptor *invites reflection* characterizes the CGI framework idea that teachers should invite students to engage with and reflect on other students' mathematical thinking. *Invites reflection* encourages students to engage with the details of another student's strategy(ies) or have other students engage with theirs. When this occurs, students are exposed to new strategies and mathematical concepts and they see that they can learn from each other.

Some clarity about differences in elements is pertinent at this point. In my coding of CGI framework, I coded *invites students to share* to mean an instance of a PSET's asking the class if anyone wished to share their strategy. *Invites students to share* is different from *broadcasting* in that broadcasting was a way for the PSET to make sure students could hear a strategy a child offered, perhaps because of a noisy room or a quiet student voice. *Invites multiple strategies* was the code I used for PSET's soliciting of different solution strategies, or variances in similar strategies.

Jennie did not incorporate *invites reflection* into her first lesson plan or first enacted lesson. She did, however, frequently integrate it into her second and third lesson plans and enacted lessons. A look at the LPI sessions' effects on Jennie's use of *invites reflections* follows.

Invites reflection in LPI sessions. In Session One, I shared with Jennie the element of *invites students to reflect* on other students' thinking. In the session I told Jennie what this element often looked like:

> They ask students to share their thinking . . . and reflecting on other students' thinking and "What are they thinking, you know?" What do you think they grabbed from what the other student said? What [did] Susie learn from Johnny? Because Johnny may have this really interesting solution strategy that you never thought of.

It is true that children often share invented versions of commonly used strategies. That is, children will often respond with different versions of strategies that might not be readily noticed and analyzed for the matching CGI strategy. For example, a child might make partial sums of a direct modeling strategy, breaking down a ten into five and four and one and still be practicing a form of direct modeling. I wanted Jennie to be aware of this and help her see the value of leveraging these natural learning opportunities so students would be situated to learn from each other. Since *invites reflection* was not an element in Jennie's first lesson plan or enacted lesson plan, I wanted to make sure she was exposed to this several times in the LPI.

In the second LPI session, *invites reflection* was discussed as Jennie and I co-wrote her second/LPI lesson plan. She was concerned about when she should share a solution strategy in the back and forth discussion. I told her that once a student shared a strategy/thinking she should:

> 1 Researcher: Have students comment on what other students said. This is how Susie solved it. What do you think about how Susie solved it?

2 Jennie: [typing] How this story problem . . . so . . . I won't even have started to talk about it quite yet, about how I would solve it?

3 Researcher: Yes.

4 Jennie: So then once I say, “How do you think so and so solved this problem?”, then will I show how I might solve it?

5 Researcher: In CGI you want multiple solution strategies, so have another student share how they solved this problem. Ask them again, “Does somebody else have a different way? Does someone want to share a different way of doing this?” Something like this. Again we are looking for multiple strategies.

6 Jennie: [typing] Student comes to the board and have some students comment on the student’s solution. Right?

The discussion shed light on how Jennie tried to blend student sharing of strategies with *invites reflection of other students’ thinking* into her instruction. She seemed to see her own direct instruction as another strategy that students should see while somehow making room for student reflections and comments of the thinking that emerged in the lesson. As Jennie perceived visual models (math mountains) to be similar to quantitative strategies (counting on from, counting on to, etc.) I thought it would be good to pause the second session to address this difference.

Invites reflection in lesson plan two and enacted lesson two. As a reminder, Jennie’s second lesson plan objectives were to solve a join, result unknown problem (4 + 5 = ___) and a join, change unknown problem (4 + __ = 9). Showing the element *invites reflection* in her second lesson plan, Jennie wrote, “Have some students comment on the student’s solution, ‘What do you think about how ___ solved the problem?’” In the next story problem activity Jennie included, “Student will comment on the child’s solution strategy”. Her formative assessment included a third instance of *invites reflection* with the question, “What do you think about how ___ solved the problem?” In her post-lesson two interview, Jennie stated that this element was new to her lesson planning experience and new to her pedagogy.

In Jennie's second enacted lesson she integrated the CGI framework element *invites reflection* six times. Her first invitation to students received no student responses, even with a five second wait time. Jennie's second invitation was responded to by a classmate who asked for the strategy to be repeated. Jennie's follow through of the *invited reflection* (concerning 8 + __ = 13) was typical for this lesson:

1 Allen:	I did a math mountain.
2 Jennie:	So Allen did a math mountain. Does anybody have a comment on what Allen just said?
3 Ryeen:	He probably counted on.
4 Jennie:	Good thinking. So Ryeen thought Allen probably counted on. Was that his [Allen's] method? Is that what you did, Allen? What number did you count on from?
5 Allen:	5.
6 Jennie:	He counted on from 5, interesting thinking. That is very exciting.

Different from her first lesson, Jennie was actively seeking student comments about other students' thinking. She was able to navigate a three-way discussion of a solution strategy, including one specific follow up question. Jennie was also familiar with the CGI framework strategy "counting on", which was an appropriate solution strategy for this problem.

Invites reflection in lesson plan three and enacted lesson three. As a reminder, Jennie's third lesson plan had three word problems in it which represented: 7 - 3 = ___; 4 + 3 + 7 = ___; and 8 + 5 + 2 = ___. Looking for the element *invites reflection*, Jennie wrote, "Teacher will have classmates comment on partner's thinking" and "What do you think about how ____ solved this story problem?" Jennie did not integrate the element of *invites reflection* in her first lesson plan, used it twice in her second lesson plan, and four times in this third lesson plan. This indicated growth in Jennie's integration of *invites reflection* as a way of *broadcasting* student's thoughts about other students' works. In an interview she stated that she did not always know what to do with some mathematical ideas, especially on the fly during a lesson. This is common for beginning teachers and difficult even for seasoned teachers.

In her third enacted lesson, Jennie demonstrated the *invites reflection* element five times, one less than in her second lesson. One vignette showed how Jennie demonstrated *invites reflection* with two students who shared their thinking:

1 Jennie:	Lee, will you come up for us? Share with us about your thinking. [child has 7 + 7 = 14 on his whiteboard]
2 Lee:	I started with a total of 7 + 7 and I got 14.
3 Jennie:	Does anyone have a comment about what Lee just did? Coopen?
4 Coopen:	Because 3 + 4 equals 7.
5 Jennie:	So he went [Jennie goes to the board] 3 plus 4 [writing this on the board] equals 7, and then he found that other 7 and he went plus 7 equals 14. Wow. [Both vertical and horizontal algorithms on the SMART Board] Interesting thinking. Now you guys are coming up with so many different ways to get to 14.

Although it was an assumption on Jennie's part how Coopen went from 7 to 14, she was attentive to both Lee and Coopen's discourses. Jennie also kept track of the quantities in the problem, going from the partial sum of 7 to the final sum of 14. Jennie partially utilized Coopen's reflections of Lee's strategies, showing her version of *invites reflection*. This was a marked difference from her first lesson and a smaller, yet legitimate, difference in her second lesson.

Changes in Jennie's use of invites reflection. Jennie went from not adopting this element at all in her first lesson plan and first enacted lesson to enacting it five times in her third lesson. I asked if the LPI sessions helped her lesson planning in this regard and she replied, “I think I more went off what I learned previously from our training session, by trying to ask them what they remembered, by commenting on other students’ thinking.” By her third lesson, Jennie seemed to have adopted the unfamiliar pedagogical strategy (inviting reflections) and was able to improve upon it in just two lessons, emerging as a novice, but successful, integrator of the *invites reflection* element.

Jennie’s use of builds on student starting points. The CGI framework strategy *builds on student starting points* characterizes the teacher move of intentionally listening to a student’s first strategy statement, then interpreting and following up on the statement regardless of correct, incorrect, or incomplete thinking or strategies. Practically speaking, this element comes after *invites students to share* and before *pursues student thinking to completion*. The goal of *builds on student starting points* is to pursue the mathematical principles the student is using in his or her strategy and follow it to its conclusion, which might be erroneous or incomplete. Since CGI framework is not a scripted pedagogy or formulaic paradigm, it can be difficult for new teachers to build on student starting points. It is also difficult to not jump in and provide a solution strategy, but to help students value their own intuitive problem solving abilities. *Builds on student starting points* also supports children’s productive struggles to learn mathematics (National Council of Teachers of Mathematics, 2014).

Builds on student starting points in lesson plan one and lesson one. Jennie did not directly address this element in her first lesson plan. However, she included formative assessment questions that, if acted upon, would elicit more mathematical thinking from students and start the *builds on* process. In her lesson plan she wrote, “How did you figure that out?” and “Can you show me your thinking?” That Jennie included these in her lesson plan before the LPI sessions shows she valued student thinking and wanted students to share their strategies. She did not include details of when she would ask these questions during the lesson.

During Enacted Lesson One, Jennie often asked students to “share your thinking” but did not practice *builds on student starting points* element in a CGI framework manner. Instead, the many discussions began with an *invite to share* their thinking, followed by one student sharing, and then Jennie would offer a short quantitative or model-type interpretation of the child’s response. Jennie’s sharing would usually be longer if the child offered an “equation” (algorithm) as part of the response about the mathematics problem.

Builds on student starting points in the LPI sessions. In our first session, I explained to Jennie the element of *builds on student starting points*. A bit later in the session I added, “So wherever students start [a problem] they can progress through it in the sense of allowing them to take the lead. So you use what they have started with, and help them to go forward from that point.” After articulating other CGI elements, I told Jennie:

> The intention is to listen to student math thinking, which you did, and demonstrate willingness to struggle identifying students’ math thinking . . . you're trying to figure out what they did to solve it, and you pause for a little bit, that's a great thing. Because you're really interested in “How did you get to 34 like that?”

Later in the first training session we viewed videos of students solving problems and discussed what *building on student thinking* can look like. Different problem types were presented so Jennie could see how the narrator navigated the discussions with an eye to matching problem types with specific solution strategies from the CGI solution strategy chart (Appendix F).

In the second LPI session, we co-created the second lesson plan with the goals of having students solve join, result unknown and join, change unknown problem types. One of the conversations we had shed light on what *builds on student starting points* might look like:

1 Researcher: If I'm understanding you, students will gather on the rug. Students will discuss how they solve problems 2 and 3 on the board and you're going to walk through that with them. And I would say after this, "You're going to help them clarify their thinking for the other students. You're trying to reveal what their thinking is. You are trying to make sure other students . . . at least hear how they" [pause . . .]

2 Jennie: Help clarify thinking for the whole class.

3 Researcher: To clarify their solution strategy is really what you are trying to look at for the whole class. You're trying to make sure did they do this, what did they do? Did they count on? Did they count from? [Did] they match? Did they pair? You're just making sure the rest of the class has an idea of what that student did. Maybe it's really clear when they shared a comment and maybe it isn't.

A bit more conversation about following through on student thinking ensued, with Jennie wanting to know more about orchestrating how sharing could be done, how modeling strategies could be organized, and what worksheets and assessments she wanted to use.

Builds on student starting points in lesson plan two and enacted lesson two. Jennie's second/co-created lesson plan incorporated *builds on student starting points* through the statement "Teacher reveal[ing] the child's thinking for the whole class. [The] student will comment on the child's solution strategy." It was Jennie's intention that following up on student starting points could be done through analyses of other's comments. For a beginning teacher to "let" other students comment on their peer's thinking was a demonstration of Jennie's faith in children's thinking capacities and of her willingness to write a lesson plan that prioritized student input while keeping the unveiling of student mathematical thinking at a high level. The LPI sessions appeared to affect Jennie's second lesson plan as she made more lesson space for children's thinking and for multiple strategies to be shared. Her second lesson plan also allowed for more student to student interactions than her first lesson plan. *Builds on student starting points* carried over into her second enacted lesson.

Jennie's second enacted lesson revealed five instances where Jennie sought to *build on student starting points*. Two of the five instances included specific follow up questions, necessary for building on student starting points. One discussion between Jennie and Jelema demonstrated Jennie's new skill of this element:

1 Jennie:	How did you figure that out?
2 Jelema:	I did 5 plus 8 = 13 [showing to class 5 + 8 = 13 on her whiteboard]
3 Jennie:	How did you figure that out?
4 Jelema:	Because I used circles and I used plus because it is adding.
5 Jennie:	Wow, that was very observant.
6 Jelema:	And it made 13.
7 Jennie:	How did you know you were adding?
8 Jelema:	Because plus means adding and minus means subtracting.

Jennie's talk turn 5 response was generally complimentary and reminiscent of her first lesson pedagogy. Her talk turn 7 response went beyond algorithmic thinking and demonstrated growth in her skill set of *builds on student thinking*. Jennie built on Jelema's first reply through general and specific follow up questions. It also demonstrated Jennie's attention to a mathematical operation that was not directly a part of this conversation, but was a connection to an earlier discussion of using addition and subtraction for different purposes. There were significant differences in *builds on student starting points* between Jennie's first and second enacted lessons.

Builds on student starting points in lesson plan three and enacted lesson three. Jennie addressed this element through her denotation of "partner sharing" where Jennie asked students to share their thinking with their partner. This was different from her first and second lesson plans, where she asked students to share with the whole group.

In the post-lesson conversation, Jennie shared that she had three sources for this lesson - her written lesson plan, her notebook of ideas about the lesson, and the LPI training sessions. Jennie shared that in the lesson "I think I more went off what I learned previously from our training session, by trying to ask them by what they remembered, by commenting on other students' thinking . . . I gave my opinion a little bit too quick." Later she recalled the value of having general follow up questions available during instruction - "I think just having those, I think that was my biggest impact from a lesson plan, was just making sure I was asking those questions."

In her enacted third lesson, Jennie made an interesting decision to model for the class what it would look like for students to share with each other - a student centered and creative way to reveal and build mathematical thinking between partners. To do this, she invited a student up front and began the conversation:

1 Jennie:	Drey has 7 pieces of candy; he gives me 3 pieces of candy, now how many pieces of candy does Drey have left? How do you think he could solve that? Show me your thinking. Allen, can you come up and show your answer? You can just bring up your whiteboard and show the class.
2 Allen:	So I figured out the problem because I used circles and then I crossed out 3 [had 7 – 3 = 4 on his board] and then I had 4. After I figured it out I wrote the equation.
3 Jennie:	Does anyone have a comment about what Allen just said?

4 Child:	How did you figure that out?
5 Jennie:	[To Allen] Tell us how you figured that out?
6 Allen:	Well, I used circles and I crossed out three of them.
7 Jennie:	He drew circles and crossed out three of them. He knew that Drey was giving me 3 pieces of candy so that was really generous of Drey. Thank you for coming up and being my example. [To the class] So does this kind of make sense then what we will be doing?

Jennie demonstrated a new pedagogy by modeling for the class how they could elicit problem solving strategies with their partners. In so doing she created an effective learning space for students to learn how they could ask questions of each other, and in so doing, reveal mathematical thinking and strategies. I was caught off guard and pleasantly surprised by her creation of this student-centered approach and how it fostered an inquiry-based form of teaching that proved effective in *building on student starting points*.

Changes in Jennie's use of builds on students' starting points. Jennie grew in her practice of *builds on students' starting points*. By her third lesson, Jennie was more intentional in asking specific follow up questions, resulting in more articulate student offerings of mathematics strategies. When asked about uncovering student thinking, Jennie replied:

> It's hard to know someone's thinking just by looking at [it]. A lot of different people can get 7 + 3 equals 10, but there are a lot of different ways that go into that, like how we talked about in the training if we do the "counting on" is less sophisticated than other methods, so knowing by how they solve different problems you can tell where they are at in either their math confidence or understanding . . . It's just nice to see what they're thinking and how they got to that because sometimes you can't always tell there's an issue with an explanation. It seems like an understanding of part of the concept is going to look a little different. And you can probably tell that by looking at their thinking.

Jennie and Research Questions One and Two

Research Question One asked, “What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?” During the study Jennie demonstrated a progressive increase in the number of CGI elements she integrated into her lesson plans. Jennie integrated 10 elements into her second lesson plan and 11 into her third, in contrast to 6 elements in her first lesson plan. It seemed evident that this increase was at least partly due to the LPI. The analysis of four elements used by Jennie (*broadcasting, invites reflection, invites multiple strategies,* and *builds on student starting points*) confirms that she integrated more instances of these four elements into her lesson plans after the LPI took place.

There is also evidence that the increase in integrated elements came partly from Jennie’s teachable disposition. Jennie had shown this characteristic in the mathematics methods course and other courses as well. If Jennie valued being trained in content and pedagogy from her instructors, she would be predictably disposed to integrate CGI elements portrayed in the LPI sessions. Her successful integration of the elements might also be due to the combination of her teachable disposition and the LPI.

Research Question Two asked, “What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?” In this study Jennie showed a substantive increase in the number of CGI elements she utilized in her enacted lessons. Of the 18 identified CGI elements, Jennie enacted 7 in her first lesson, 13 in her second, and 11 in her third. Given this increase it was plausible that the LPI sessions were influential in Jennie’s decision to increase her use of elements to elicit student mathematical thinking. It was notable that Jennie was successful in leveraging many of the 18 elements as early as the second lesson (13) and that this continued into her third enacted lesson (11). When asked about what the CGI framework-based LPI did for her she stated,

> I liked it. I thought it was good. It is good to have a refresher, there are so many things, like I want to do the best for my students so I feel like right now I don't feel super prepared so I think it's good that I'm getting a refresher.

Jennie's enactments of the elements were situated in problem solving contexts in all of her lessons, which is the intent of CGI theory and practice. However, in her first lesson she pursued only equations as solutions, which are just one type of representation. The LPI appeared to have opened Jennie's perception that utilizing CGI elements in her instruction could help her elicit and interpret other representations of student mathematical thinking - visual, tactile, oratory - as these are legitimate avenues students will use to explore and communicate mathematical information.

Jennie and Research Question Three

Research Question Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" Jennie's teaching practices in her three enacted lessons were recorded and analyzed for associations with CGI elements. Before the LPI, Jennie's instruction was mostly direct instruction, using "math mountains" and "equations" (actually mathematical expressions) to find sums in the teens. She integrated fewer CGI elements into her first lesson plan and enacted lesson than after the LPI. In her first enacted lesson, she spent most of her time speaking for students, *broadcasting* students' first responses to finding sums and then speaking for students as she explained counting on strategies. Some literature addresses this as "jumping in" to discourses.

It appeared that Jennie demonstrated in her first lesson a form of CGI instruction without the depth she would demonstrate later in the study. For instance, her *broadcasting* was mostly of her own thinking and not that of her students. Her *posing of problems without modeling* gave potential learning spaces for students, but she spoke for them instead of letting them draw out their own thinking. She *invited multiple strategies,* but limited students to symbolic representations (6 + 7, 4 + 9). Jennie was essentially applying a few CGI elements as best she could remember from the previous methods course. Her instructional shortcomings pointed to the need for this LPI study, designed to rebuild and reinforce Jennie's capacity to elicit student thinking.

Changes in Jennie's Teaching Practices

Jennie's second and third enacted lessons demonstrated greater use of CGI elements, mostly through the teaching practices of directed discussion and inquiry-guided discussion. In her second lesson she practiced directed discussion while she *built on student starting points*, followed through with specific questions, and *listened intentionally* to thinking. In her third lesson Jennie practiced inquiry-guided discussion when she *invited students to reflect* on other students' thinking. For example, she asked students four times if they wanted to comment on another student's responses. Her inquiry guided discussion strategy elicited student strategies for making tens from two addends (partial sums) and then adding the third addend to the total.

A second type of instructional change emerged as she moved toward more student-centered discourses, with students sharing more of their mathematical thinking in her second and third lessons than in the first lesson. These discourses utilized CGI elements like *inviting students to reflect* on other students' thinking, associating CGI elements with directed discussion pedagogy. Jennie clearly changed her instructional practices from mostly direct instruction and speaking for students to practices that were more effective at revealing student strategic thinking. In her third lesson, as she diverted from her previous instructional practice of speaking for students, Jennie was intentional about utilizing guided inquiry discussions to elicit and broadcast students' ideas about constructing partial sums to 10 and then adding the third addend to achieve the sum.

It should not be concluded that this LPI study had any direct effect on Jennie's lesson discourses in general, but could be said that Jennie's increased use of CGI elements in her second and third lessons was associated with an increase in student-teacher discourses that elicited more mathematical content than in her first enacted lesson. Using specific elements helped Jennie better elicit and reveal student mathematical thinking than before she had the LPI. Both the quantity of mathematical strategies elicited and the depth of the discussions changed for Jennie. She was willing to ask more open-ended questions and was much slower to interject her own mathematical thoughts/strategies into the vignettes she was part of. Jennie continued to give direct instruction, especially when students were off track mathematically, but her "jumping in" occurrences were after students spoke, and not before. Students also seemed to be freer to ask general questions of other students' strategies after the LPI, modeling the element of *inviting students to reflect* on other students' thinking. This freedom was evidenced in more students "jumping in" to Jennie's discourses, even if just to ask "How did you get that? You didn't do any drawing."

Other variables could have come into play here, such as more time discussing CGI principles with the researcher, more comfortability with being observed, less ambiguity about how a learning framework can be realistically integrated into lesson planning, and other factors.

Penny as a Case

Penny presented a unique case for this study. She was completing her bachelor's degree in education along with licensure programs in elementary education and middle school communication arts (fifth through eighth grades). Neither Jennie nor Eleanor had this combination of study and licensure. It was hoped that this would situate Penny as having different skill-sets, experiences and/or expectations for this CGI framework-based LPI study. Penny was married and in her early twenties.

Penny's first survey responses revealed that her recollection of CGI elements from the previous mathematics methods course was somewhat fragmented, remembering general categories of problem types and some solution strategies. This was commensurate with Jennie and Eleanor's recollections.

On the pre-assessment informal survey, Penny revealed her thoughts about her mathematics experiences and feelings. When Penny was young, she had a favorite mathematics teacher who "did not stop until she had explained everything that she could until I could grasp the concepts. She encouraged me in the tiny things." Penny also shared her feelings about her working relationship with mathematics:

> It's difficult for me to grasp certain concepts – I'm learning to have a better relationship with it. As of last year I used to process in my head for it and trying to change my mindset. I'm not very confident in my ability to do fractions, so cooking or taking measurements can be stressful. Changing things into percentages is also tricky for me so it can make shopping and leaving tips nerve wracking. I'm currently trying to reteach myself. I have a great perspective on students that are struggling. I know I will have to work the hardest in this area to assure her [cooperating teacher] that I'm equipped to teach. I'm excited that it's

> pushing me to grow and be better at mathematics. I believe I can teach math. I need to stay ahead but I have the ability to do it.

Penny's responses to the surveys, as well as my experiences with her in our mathematics methods coursework, gave me some ideas about how an LPI might increase her understanding and valuing of eliciting student thinking to build number sense. One of these ideas was to make our LPI sessions a positive experience where Penny could recapture mathematics as being conceptual and not just procedural, and that using some CGI elements would help her elicit and interpret student thinking in problem solving lessons.

The following sections highlight Penny's integration of four CGI elements into her lesson plans, LPI sessions, and enacted lessons as they occurred in an authentic learning environment. The four elements were: (a) *broadcasting,* (b) *invites multiple strategies,* (c) *invites reflections,* and (d) *willing to struggle to understand student thinking.* Through Penny's use of these four elements, data emerged that helped me understand how she utilized CGI practices as a tool and framework to help her elicit, interpret and utilize student thinking to develop her first graders' number sense.

Penny's utilization of CGI elements.

Table 4

Elements of CGI framework in ***Penny's Lesson Plans.*** *Each row is the same CGI element. There were 18 CGI elements in this study. The four highlighted elements were the focus elements of analysis in Penny's work.*

Pre-intervention Lesson Plan (1)	Intervention Lesson Plan (2)	**Post-**Intervention Lesson Plan (3)
Expects students to pursue solutions strategies	Expects students to pursue solutions strategies	Expects students to pursue solutions strategies
Presents problems without modeling solution strategies	Presents problems without modeling solution strategies	Presents problems without modeling solution strategies
Invites students to share solution strategies	Invites students to share solutions strategies	Invites children to share solutions strategies
Teachers learn after listening to students	Teachers learn after listening to students	
Builds on student starting points		Builds on student starting points
Uses students intuitive problem-solving abilities		
Broadcasts students' thinking to class		Broadcasts students' thinking to class
7 elements ↑	Uses CGI framework problem types while planning	Uses CGI framework problem types while planning
	Uses word problems to build number sense	Uses word problems to build number sense
	Invites students to reflect on other students' thinking	Invites students to reflect on other students' thinking
	Teacher willing to struggle to understand students' strategies	Teacher willing to struggle to understand students' strategies
	Invites multiple strategies	Invites multiple strategies
	Intentional listening to student math thinking	Pursue thinking to completion
	10 elements ↑	11 elements ↑

Table 5

*Elements of CGI framework in **Penny's Enacted Lessons**. Each row is the same element. There were 18 CGI elements in this study. The four highlighted elements were the focus elements of analysis in Penny's work.*

First Lesson	Second Lesson	Third Lesson
Expects students to pursue solution strategies	Expects students to pursue solution strategies	Expects students to pursue solution strategies
Invites students to share solution strategies	Invites students to share solution strategies	Invites students to share solution strategies
Broadcasting	Broadcasting	Broadcasting
Builds on student starting points	Builds on student starting points	Builds on student starting points
Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities
Pursues student thinking to completion	Pursues student thinking to completion	
Children showing respect for others' thinking	Children showing respect for others' thinking	
	Uses CGI framework problem types	
	Invites students to reflect on other student's thinking	
	Uses word problems to build number sense	
	Teacher learns after listening	
	Invites multiple strategies	Invites multiple strategies
	Presents problems without modeling	Presents problems without modeling
	Willing to struggle to understand student's thinking	Willing to struggle to understand student's thinking
	Intentional listening to student thinking	Intentional listening to student thinking

Penny's use of broadcasting. As stated earlier, *broadcasting* is my descriptor for the CGI teaching practice of making a student's statements, questions, or reflections known to the whole group. When it occurs in a lesson, *broadcasting* often follows *inviting to share* and precedes *pursues student thinking to completion.*

Broadcasting in lesson plan one and enacted lesson one. Penny's first lesson plan focused on writing numbers in expanded form. There were no story problems present in the plan. Her first lesson objective was for students to work toward number name recognition. Penny's second lesson objective asked students to "practice writing numbers in expanded notation". Her third lesson objective stated, "I can write two digit numbers in expanded form, which looks like an equation. 40 + 6 = 46." As it is difficult to integrate CGI elements into lessons designed to be procedural, it would have been problematic for Penny to have tried to do so. CGI theory and practice was designed to help teachers elicit student thinking when problem solving, so integrating elements into Penny's lesson would be understandably unsuccessful. Three students volunteered to share their thinking during the lesson.

Penny included a form of *broadcasting* in her first lesson plan through asking students to come to the SMART Board and pair two digit expressions (30 + 2) for the class to see. She also included *broadcasting* in her assessment section of the lesson plan with the phrase "having students come forward to solve various problems." She did not include instructions for *broadcasting* student thinking/sharing during the lesson.

In her enacted first lesson, Penny *broadcasted* many student responses to her invitations to share. She repeats students' responses to naming a two digit number, some being incorrect responses. Students were clearly able to hear what all their classmates said because of Penny's *broadcasting* of their responses. Penny was consistent about *broadcasting* students' initial statements.

Broadcasting in LPI sessions. Like the first LPI session with Jennie, Penny's and my first session discussed all 18 CGI elements and what they look like when teaching a lesson. The session also covered levels of sophistication in mathematical problem solving, word problem types (Appendix E) and how Penny could keep track of these using the children's solution strategies chart (Appendix F). *Broadcasting* was discussed from students' perspectives when Penny related, "We were having kids talk about their thinking and then asking other students what they thought". When lower level skills are the focus of a conversation, which can occur when procedural/algorithmic thinking is occurring, less cognition is required and there are fewer conceptual connections being leveraged.

In the second LPI session we discussed Penny's lesson objectives and the mathematics behind the problem types she wanted to teach to the students. As we built her lesson, my instruction about broadcasting (I did not use this term) was focused on revealing student thinking. I told Penny,

> You'll put that up on the board and ask "Did anybody else solve this problem a different way?" Then give them time to do that, and then you'll say, "I think you did this . . . " just so the other students can see this is what they did. "Do you guys have any thoughts about that?" Just to see what they say not really looking for confirmation, you really want to know what they think. One person may say, "How did you do that?" Then you go back to that person and say, "The student wants to know how you solved this? Can you do that again?"

We did not spend much time looking at how to follow through with students' initial strategies (*builds on student starting points*) but making sure students would have as many opportunities as possible to share their initial thinking about a problem and that their responses would be available (auditory at least) for their classmates' consumption. In Chapter Five I share more on how the LPI could have better addressed further pursuit of students' initial offerings of their mathematical thinking.

Broadcasting in lesson plan two and enacted lesson two. In Penny's second/co-constructed lesson plan the objectives were, "Students will add 2 digit and 1 digit numbers by counting on" and "Students will solve 'join, result unknown' and 'join, change unknown' word problems". Nine students volunteered to share their thinking during the lesson. *Broadcasting* was indirectly referenced in her scripted instructions with, "When sharing their answer, model on the board what the student is doing. Keep asking clarifying questions. I am looking for clarifying questions. I am looking for clear articulation of their strategies." This was written four times in the plan, once for each of the four word problems. The second LPI session articulated this idea and helped incorporate *broadcasting* into her lesson plan.

Broadcasting was more prominent in Penny's second lesson plan than in her first. She was in a better stance to integrate *broadcasting* into her enacted second lesson than in her first because of the LPI sessions. Penny was also in a better stance to leverage *broadcasting* because she had a focus on pursuing multiple solution strategies (not just equations) as students were guided through word problems. Penny's second enacted lesson illustrated *broadcasting* in seven vignettes. Most of these followed a pattern: (a) Penny invited a student to share, (b) student shared strategy, (c) Penny repeated student's strategy, (d) Penny asked one follow up question, (e) student answered question, and (f) Penny finished mathematics strategy. In one particular discussion, Penny did more *broadcasting* than in the rest of her lesson. This was evidence of using *broadcasting* for more than a general share-all time and that Penny was beginning to use *broadcasting* to elicit more detailed mathematical thinking for the class to see. A vignette with Fifi is exemplary of Penny's using *broadcasting* to make sure the class knew what a student was saying:

1 Penny:	Any other thoughts about how Gretta did it? Fifi?
2 Fifi:	Um, she took 2 blocks and 2 blocks and counted them all up and her total was 5.
3 Penny:	You said 2 blocks and 2 blocks but I see you have 3 and 2 blocks. Did you mean a 3 and a 2?
4 Child:	Yes.

Penny made sure that Fifi's response was correctly interpreted and that the rest of the class could follow Fifi's strategy at this point.

Broadcasting in lesson plan three and enacted lesson three. For familiarity, Penny's third lesson plan objectives were the same as her second lesson objectives - that students would add one and two digit numbers by "counting on" and that students would solve join, result unknown and join, change unknown problem types. Since her objectives mentioned only one of three common solution strategies for these kinds of problems, it was evident that Penny had not anticipated addressing multiple strategies and the conversations that would emerge from these other strategies. During the lesson, six students volunteered to share their thinking. In her third lesson plan, Penny added a thought not present in her first two lesson plans when she wrote, "I will put an emphasis on the process they went through rather than the strategy itself. Were they able to *count on* from the number?" Broadcasting occurred in Penny's third lesson plan through the same wording as in her second plan. She wrote, "When sharing their answer, [I will] model on the board what the student is doing. Keep asking clarifying questions. I am looking for clarifying questions. I am looking for clear articulation of their strategies." She was beginning to see how broadcasting could unveil student thinking for other students to ponder.

Broadcasting in Penny's third enacted lesson often consisted of Penny repeating a student's initial strategy response and then concluding with her own reflection of what she thought the student's strategy was. Penny's exchange with Lele is representative of her pedagogy in the third lesson:

1 Penny:	Lele, do you want to come up and show us the way that you solved it [3 + __ = 10]?
2 Lele:	[drew 3 squares under the 3 addend and 10 more squares under the blank addend]
3 Penny:	So you drew 3 and then 10.
4 Lele:	So I got 13 [writing 13 in the missing addend place]
5 Penny:	You got 13. So you added these two numbers [pointing to the 3 and 13] hmmm. Anybody have any thoughts on that?

While Penny did not follow through on Lele's misconception for this missing addend problem, Penny did not jump in with an answer as she did in her first lesson. Also, Penny did clarify Lele's strategy for the whole class to observe, and then asked others to reflect on Lele's erroneous solution. This was a big change from her first lesson and a small change from her second lesson. Penny was demonstrating a tendency to not offer a correct solution and broadcasting student's thinking even though it was erroneous. Both of these showed growth in Penny's willingness to broadcast student thinking.

Changes in Penny's use of broadcasting. Differences in *broadcasting* existed between Penny's enacted lessons. In her second enacted lesson, Penny tended to terminate discourses where students were inaccurate by asking another student to share.

This, in a sense, increased her *broadcasting* via more sharing, but did not unpack student mathematical thinking as it could have. In her third enacted lesson, Penny integrated *broadcasting* by doing more direct instruction of solution strategies, especially when students revealed erroneous strategies. In her third lesson she often grasped students' initial incorrect responses and diverted the conversation by sharing her solution strategies in length. In one instance Penny broadcast an incorrect solution given by a student. It took Penny a moment to realize the error in the child's strategy, and Penny addressed the error as a mistake to the class. In this instance Penny did not take the time to pursue the student's thinking, bringing it to a conclusion that would have been contrary to a previous correct strategy. As a missed opportunity to compare two solution strategies, Penny could have explored this common misconception with the class. It may have been Penny's comfort with equation-based solutions that affected an increase in her *broadcasting* of her own thinking to the class. Another factor that may have increased Penny's use of *broadcasting* was her preference of direct instruction as an efficient and accurate method of delivering mathematics content. For whatever reason, Penny's use of the CGI element of *broadcasting* increased.

Penny's use of invites multiple strategies. As a reminder, the descriptor *invites multiple strategies* characterizes a teacher's act of inviting students to share a different solution (or perceived different solution) strategy(ies) than one shared previously. The comparison of strategies helps reveal mathematical patterns and opens doors for students to see their own math thinking in new ways.

Invites multiple strategies in lesson plan one and enacted lesson one. Penny's first lesson plan did not implement the CGI element of *invites multiple strategies.* This is understandable as her lesson's activities did not have word problems as the focus of her instruction.

Penny's enacted first lesson did not reveal Penny asking students for multiple strategies. However, she did ask six students to share their thinking about how to pronounce one and two digit addition expressions (20 + 3 as 23). In the post-lesson conversation, Penny mentioned that she did not challenge the students mathematically, but that it was a review lesson. She stated that the lesson "was more of a review day, they weren't seeing anything new, they saw something similar to last time. It was really a matter of making the concept concrete for them. So it wasn't in depth; I'm sure it didn't fully challenge them." Penny could have expanded students' thinking about these mathematical expressions by revealing multiple translations/representations of these quantities through frameworks like Lesh's Translation Model (Lesh & Doerr, 2003).

Invites multiple strategies in LPI sessions. In the first session, I shared that Penny should look for different kinds of strategies from students. We reviewed the strategies for CGI framework problem types and how the levels of sophistication were accounted for on the children's solution strategy chart (Appendix F). Looking at the chart, Penny mentioned:

> I can tell you already this makes way more sense than last year. I was almost in tears thinking this was overwhelming for me when I hadn't even taught these types of things yet, but now that I have taught problems that look like these pretty regularly to my students, some of those easier ones that we've gotten into, it makes sense where they're at, and how they're there and why they are there.

During the second LPI session, Penny and I integrated *invites multiple strategies* into her lesson plan. My instruction was,

> You'll put that [a solution strategy] up on the board and ask, “Did anybody else solve this problem a different way?” Then give them time to do that, and then you'll say, “I think you did this” just so the other students can see what they did. Ask “Do you guys have any thoughts about that?” just to see what they say, not really looking for confirmation. You really want to know what they think.

Invites multiple strategies in lesson plan two and enacted lesson two. During the second LPI session Penny and I integrated *invites multiple strategies* four times into the second lesson plan using the phrase, “Would someone else like to share what they did?” Penny and I did not include phrases similar to, “Does someone have a different way to solve this?” Penny also included the phrase, “Use follow up questions, maybe asking them to rephrase,” four times in the lesson plan - once for each story problem. This next step was to pursue multiple strategies for the class to examine so that concepts underlying strategies could be explored and compared. As Penny did not include the idea of *invites multiple strategies* into her first lesson plan, this was a big change in her thinking about the math content of the lesson.

In her second enacted lesson, Penny asked students six times if they had a different strategy for a word problem. This was a big change from her lesson plan and from her first lesson. I was not sure why she spoke this phrase when it was not in her lesson plan, but it appeared to have come from the LPI emphasis of asking students to share their strategies and for teachers to intentionally look for multiple solution strategies.

Invites multiple strategies in lesson plan three and enacted lesson three. Penny’s third lesson plan did not ask students to share “different” strategies, but invited students many times with the phrase, “How did you solve this problem?” Penny’s lesson plan included multiple opportunities for students to comment on other students’ strategies and thinking. These opportunities occurred through the problem types she selected: join, result unknown and join, change unknown. Join, result unknown problems are commonly solved with three different strategies, and join, change unknown are commonly solved with two different strategies. Penny's selection of these two word problem types opened many possibilities for Penny to elicit and interpret students' interactions with the mathematics behind the five strategies.

Penny *invited multiple strategies* from her students two times in her third enacted lesson, despite not including this in her third lesson plan. This was less *invites* than in her second lesson, but more than her first (which had no *invites multiple strategies*). As in her second enacted lesson, Penny often invited students to share their thinking and comment on other students' strategies. I believe the decrease in *invited multiple strategies* in her third enacted lesson was due partly to her not using story problems in it, choosing to use equations instead. I was not sure why she chose *not* to read the two story problems from her lesson plan to the class, but believe it was her thoughts about children's reading that sometimes complicated the mathematics of a problem. Penny explained it like this:

> I think you would think word problems might be easier cuz it's a more natural way of looking at math. Instead, someone is not going to come up to you and saying, "Hey, what's 6 minus seven" they're going to ask you in such a phrase like, "I only have seven of these and someone took six of them, how much do I have now?" That would be a more natural way to hear a math problem, I would think, but for some reason I don't know if it's because we don't always start with word problems kids don't know what it looks like in kindergarten exactly, but for some reason the actual numbers in an equation seem to be more comfortable for kids. And when you add a word problem to it, it could be elements of reading comprehension, it could be attention spans of listening to the whole thing closely enough, but something in all those words tends to get some of the kids a little bit lost. I don't know, personally, word problems intimidate me more than equations, and I don't know if it's because you have to weed through the word problems to find what you're actually doing, it doesn't straight up tell you are you adding, or subtracting, is something missing? You have to weed through the words to try to translate what you are actually looking to do. That could be an element, too.

It makes sense that Penny would choose to use equations (symbolic representations) in her third enacted lesson with this belief structure. In this lesson Penny did more direct instruction than in her second lesson, much of it adding to the context of an equation, which a word problem could have provided originally. Many PSETs have inadequate problem solving skills that manifest themselves in content and procedural knowledge weaknesses (Taplin, 1998).

Changes in Penny's use of invites multiple strategies. With no *invites* in her first lesson plan or first enacted lesson, Penny clearly integrated *invites multiple strategies* more often in her second and third lessons. Especially in her second enacted lesson, Penny was intentional and successful in *inviting multiple strategies* from her students. Penny's pedagogy had changed such that she had begun to prioritize the elicitation of students' varied solution strategies, making them available for other students to consider and for her to leverage for mathematical sense making. In her second enacted lesson Penny asked, "Did anyone else solve it a different way? It's ok to use blocks. Did you use a different way?" She repeated these sentences a second time a bit later in the lesson. While she may have considered the use of manipulatives (physical representations) as another way of solving a word problem, Penny did not discern the different mathematical sophistication levels that can emerge by the use of the same representations. In other words, Penny did not seem to recognize that manipulatives can be used by children to show different levels of mathematics understanding, which the CGI framework provides for a teacher. For instance, the same manipulatives/counters can be used to show a direct modeling strategy or a more sophisticated counting strategy. Penny did not seem to understand how a learning tool or representation is different than the mathematical concepts that undergirded a strategy– at least in the *in the moment* opportunities that happened while she was teaching.

Penny's use of invites reflection. As a reminder, the descriptor *invites reflection* characterizes the CGI element that teachers should invite students to engage with and reflect on other students' mathematical thinking. These reflections elicit important mathematical ideas that can be leveraged to build students' number sense.

Invites reflection in lesson plan one and enacted lesson one. Penny's first lesson plan did not include *invites reflection*, which was not unexpected as the lesson did not include story problem solving activities. Her lesson plan did include the use of manipulatives and small group work on expanded notation.

In her enacted first lesson, Penny did not ask students to reflect or comment on other students' thinking. Her primary investigation was about mathematical expressions with tens and ones. She pursued student thinking with specific follow up questions and helped them engage in place value concepts.

Invites reflection in LPI sessions. *Invites reflection* was discussed twice in the first LPI session. Penny discussed how a lesson taught the previous day, with her cooperating teacher, enacted *invites reflection* and how excited she was to see her cooperating teacher practice this element. Later in the session I stated twice that inviting students to reflect on other students' thinking is recognized by CGI research as effective practice.

During the second session, as we were co-constructing the lesson plan, I discussed several times what *invites reflection* might look like:

> This [student] has shown you a strategy, and you have made it clear what this person did, and then you can ask them "What do you think?" So [you're] just

> trying to orchestrate this. Now the student has demonstrated the problem, and the class has seen that. Now you can ask the class "What do you think about how Susie solved that problem? What do you think about that?"

My intention for helping Penny incorporate *invites reflection* was to include more students in conversations, allowing for more strategies of mathematics to unfold. I was also looking for learning spaces where comparisons of strategies could be examined (Chapin et al., 2013). I did not discuss or review with Penny how she could orchestrate such comparative discussions, something I address further in Chapter Five.

Invites reflection in lesson plan two and enacted lesson two. In each of the four planned word problems, Penny included the phrase, "Ask the class, 'What do you think about how ____ solved the problem? What do you think about the way they solved the problem?'" This was a big change from her first lesson plan, where Penny did not offer invitations to reflect. Her four *invitations to reflect* demonstrated Penny's belief that this would help her observe student thinking, an important step in Penny's understanding of the mathematics behind their responses.

In Enacted Lesson Two, one vignette was representative of the two occurrences of *inviting reflections,* where Penny told her class:

> And then [the student] counted those all up. So I'm going to draw that. She had one cube, two cubes, three cubes, and she had another cube and another cube [5 cubes drawn on the SMART Board]. And that's how she solved it. Friends, how do you think about how she solved it? What are your thoughts? Does anyone have any thoughts on how she solved it?

Penny was partially successful at *inviting students to reflect* on other students' responses in her second lesson. There were two instances where she initiated the element in her second lesson. The two instances led to a conversation about the "counting on" strategy and a clarification of a visual explanation. The LPI seemed to directly affect

Penny's capacity to invite students to reflect on other students' mathematical strategies. While Penny made advances in her ability to orchestrate this element, she struggled with how she should pursue student reflections:

> It was hard to gauge how far I should pursue. I think that I did, thinking about what I did, I tried very hard to ask why they did it. And I did ask what did they think about what that other person did? That's not a question I think I would normally ask. It doesn't seem natural to ask others what they think of others' thinking and I think it threw the kids off for a little loop, but I would say that I did a fair job asking and pursuing them.

Invites reflection in lesson plan three and enacted lesson three. Lesson Plan three records the same *invites reflection* phrase used in the second lesson plan, "What do you think about how _____ solved the problem?" After this, Penny gave space for another strategy to be shared, followed by another *invitation to reflect.* I did not follow up on whether Penny intended to have reflections on reflections or just reflections on original responses. The LPI appeared to have established the *invites to reflect* element into Penny's lesson planning repertoire for eliciting student mathematical thinking.

The element *invites reflection* occurs twice in Penny's third enacted lesson, the same number used in her second enacted lesson. Penny taught the lesson using the *invites reflection* element even though she did not teach any word problems. The first instance of *invites reflection* led to a student's truncated answer about 3 + ____ = 10 using the "counting on" strategy. The second instance helped a different student expose an erroneous solution given by the first student, who incorrectly added the 3 and the 10. Penny was able to elaborate on the second student's correction, building on the class' experience of solving a missing addend problem. After Fifi's initial dialogue, it continued as follows:

1 Penny: The problem doesn't quite read . . . 3 + 10 equals 13. If it did, it would equal 13, but it's telling us that it is equal to 10. We already know what it equals. Using Fifi's method with the blocks what other way could we use to solve it to find the answer? Jaden?

[pause] We know that it's not 13, so I don't want to confuse our brains, so I will erase that [erased the 10 in the missing addends spot] so we know that we have 10 blocks. 3 + what . . . [Penny holding 10 yellow blocks and three red blocks in her hand for the class to see] How can we figure out what the box is [the missing addend box]? Aubrey?

2 Aubrey: It equals 13.

3 Penny: Yes, if we added these it would equal 13. Okay, so what Bob did, he counted up to 10 and found out that the number in here [pointing to the missing addend box] was seven. So the missing number here is 7. If you didn't get that that's okay because we are exploring.

If Penny had not *invited students to reflect* on another student's solution strategy, the error would have gone unresolved. This was evidence that the LPI appeared to help Penny integrate the *invites reflection* element (along with *broadcasting)* to further mathematical thinking for sense making.

Changes in Penny's use of invites reflection. Penny's integration of *invites reflection* grew from zero instances in her first lesson plan and first enacted lesson to multiple instances later on in the study. It seemed the LPI was instrumental in incorporating the element into both her second and third lesson plans, as well as her second and third enacted lessons. It is interesting that Penny utilized *invites reflection* in her third enacted lesson despite her not using word problems in the lesson. The LPI sessions may have influenced Penny to leverage the practice of *invites reflection* to investigate other translations of mathematical thought, in this case symbolic translations (3 + ___ = 10). If this was the case, Penny exceeded the expectations of the study, designed only to utilize CGI elements for unpacking student thinking about word problems.

Penny's use of willing to struggle. *Willing to struggle* characterizes a PSET's action of unpacking students' mathematical reasoning through general and specific follow up questioning, for the purposes of understanding a child's thinking in detail. There is intentional focus on discovering the mathematical strategies and sophistication levels the child is using to solve the posed problem. *Willing to struggle* can be difficult pedagogy, as PSETs must resist the temptation to interject their own strategy(ies) while interpreting the sometimes confusing or incomplete statements the student offers. This interpreting is viewed through the lens that sees children as capable problem solvers, able to use their intuitive knowledge of real life experiences to solve quantitative problems.

Willing to struggle in lesson plan one and enacted lesson one. Penny's first lesson plan did not directly address this element. She emphasized direct instruction of expanded form language (40 plus 1 makes 41) through oral and written practice as well as using manipulatives if students so chose.

Penny's first enacted lesson pedagogy was cyclic: Penny posing a choice of mathematical expressions on the SMART Board, asking a student one follow up question, and finishing with direct instruction. Later in the lesson, in one-on-one sessions with students, Penny was more willing to struggle to *understand students' thinking*. The following one-on-one session was typical of her pedagogy:

1 Penny:	[helping a child one on one] So you said there were how many here? [pointing to 3 tens]
2 Student:	30.
3 Penny:	Absolutely, 30. And how many here? [pointing to 7 ones]
4 Student:	7.
5 Penny:	So if you have 30 plus 7. What is that going to equal?

6 Student:	67.
7 Penny:	67? But you just said we had 30. Let's do it this way, ready with me 10, 20, 30 [pointing to rods of 10]
8 Student:	10, 20, 30.
9 Penny:	30, 31, 32 [pointing to the ones, child mimicking Penny's voice] 33, 34, 35, 36, 37.
10 Student:	37.
11 Penny:	37.
12 Student:	[writing 37 on the worksheet]

Penny's discussion demonstrated a limited *willingness to struggle* to understand this student's thinking. Her three follow up questions were specific and not overly leading. She did not address the mathematics at hand at a conceptual level.

Willing to struggle in LPI sessions. In the first session I taught the *willing to struggle* element alongside another element, *teachers' intentional listening to student math thinking*. I shared:

> Teachers listen to student math thinking. Teachers demonstrate *willingness to struggle* with *identifying students' math thinking*. In other words, we will take the time to figure out . . . what is the student thinking? How are you going to do that? What is it going to look like? Are you going . . . to take the time to pursue that, the child's thinking?

In the first session with Penny, my intent was to reinforce the earlier mathematics methods course discussion about modeling productive struggle for students. In the course we also discussed how the pedagogy of exploring children's mathematical thinking is not learned overnight, but must be studied and improved through real-life interactions with students. This is in keeping with National Council of Teachers of Mathematics' (2014) idea of productive struggle for students as well as teachers.

In the second LPI session, I stated, "You're open to lots of different kinds of ways that they solve it. You really listen to them. You're *willing to struggle* to identify what they are thinking about." We also discussed the benefits of asking specific follow up questions to elicit their thinking such as, "What did they do after that? Did they count on from 3, or did they count 1, 2, 3, 4, 5? Did they just count them all?"

Willing to struggle in lesson plan two and enacted lesson two. Penny's second lesson plan did not articulate *willing to struggle*, but included scripted questions she would ask students after she posed each word problem. Her notes to self included, "I am looking for clear articulation of their strategy", and "I want to follow up on their thinking, using follow up questions, maybe ask them to rephrase".

Penny demonstrated *willing to struggle* three times in her second enacted lesson. The degree she pursued student thinking in these vignettes varied, but all of the occurrences elicited more student thinking than her first enacted lesson. Penny's most thorough use of *willing to struggle* was a discussion about a word problem for 12 + 6 = ____:

1 Jackson:	I counted in circles [making little circles on his worksheet each representing a single unit].
2 Penny:	You counted in circles? What did you do first?
3 Jackson:	I counted circles.
4 Penny:	How many circles?
5 Jackson:	Some circles.
6 Penny:	So you got that total. Did you just know to draw that many or did you start with a certain number of circles?
7 Jackson:	Circles.
8 Penny:	You started with a certain number of circles? What number did you start with?
9 Jackson:	12.
10 Penny:	So Jackson started with [to class] eyes up on the board, he drew 12 circles 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 [Penny drawing 12 circles on board] then what did you do next?
11 Jackson:	I counted to 6.
12 Penny:	You counted to 6. Did you just count to 6 or did you show that in some way on your piece of paper?

13 Jackson:	Paper.
14 Penny:	I see that you drew 6 more circles. Does that seem right? He drew 6 more circles [Penny drawing 6 more circles on the SMART Board]. Then what did you do?
15 Jackson:	I counted them.
16 Penny:	He counted them.
17Jackson:	And I know the number.
18 Penny:	And he knows the number [starting from the original 12 circles], he went 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. So that's one way we solve it.

In this discussion, Penny asked five follow up questions before she was able to grasp what Jackson did to solve 12 + 6. Her careful listening and specific follow up questions (*builds on student starting points*) demonstrated her *willingness to struggle*, and gave Penny a lot of detail about Jackson's level of mathematical sophistication. Her activation of this element situated herself to leverage Jackson's thinking for his own growth and for others as well. Compared to her first lesson, the LPI sessions seemed to have helped Penny not jump in during her pursuit of a student's mathematical strategy.

In our post-lesson conversation, Penny discussed a conflict she had about how to enact the *willing to struggle* element. Penny related her dilemma:

> I would say it [the LPI lesson plan] helped give me a foundation of what I needed to do, so I knew in my head that my job was to pursue and to not tell, could also confuse me at the end when they were all getting it wrong. That there was a gap in the training that I didn't know what to do with. I was kind of like I know I'm supposed to help them explore, but what if they're all exploring wrong? Like every one of them is getting like 15, 20, 16, like completely . . . wrong numbers for one, even if they were adding wrong numbers so that part was really frustrating to me. Because I wanted to help them, but I didn't know if I should give him the answer, or give them a suggestion, tell them that it wasn't adding, I didn't know if I should give them a little, give them a lot, or let it lie. And that part was frustrating for me.

It is common for teachers to struggle with what to do with errors and misconceptions while fostering students' intuitive problem capacities. In many ways teachers' *willingness to struggle to understand student thinking* mimics the struggle students have to understand their own thinking. It makes for a natural place for students to see teachers as learners and for teachers to experience for themselves what struggle feels like. PSETs need to be reflexive in their teaching practice to better understand why students struggle, including what the mathematical concepts behind their struggles may be. They also need to be flexible in their instructional approaches to meet these needs, providing places for misconceptions to be explored and corrected.

Willing to struggle in lesson plan three and enacted lesson three. In her third lesson plan, Penny asked students to engage in important problem solving activities. The plan did not use the phrase "willing to struggle" but does include, "I will put an emphasis on the process they went through rather than what the strategy itself was". This emphasis was a change from her direct instruction approach evident in her first lesson plan. The LPI sessions seemed to influence Penny's approach to lesson planning in that her second and third lesson plans were more discourse oriented and inquiry based.

Willing to struggle to understand student thinking occurred in Penny's enacted third lesson. Some occurrences were truncated by Penny, but some were more thorough, demonstrating Penny's *willingness to struggle* to understand student thinking. One discussion about a missing addend problem [__ + 6 = 12] elicited the following:

1 Penny:	Cassie is going to share something. Can you hold your blocks up and show everyone?
2 Cassie:	[Cassie holding 12 yellow blocks and 6 red blocks next to each other].
3 Penny:	Tell them how many yellow blocks you have.
4 Cassie:	I got 12 yellow blocks and 1, 2, 3, 4, 5, 6 and 7, 8, 9, 10, 11, 12

5 Penny: Yes [Penny took the 12 yellow blocks and 6 red blocks from the child and held them up for the class to see]. You counted 6 and you stopped. Why did you stop?

6 Cassie: Because that was 12 [pointing to the 12 yellow blocks],

7 Penny: Oooh,

8 Cassie: And there's 6 more blocks [pointing to the red],

9 Penny: So there are 6 more left. So she had 2 [stacks] and she knows she already has 6 even if you broke them up [separating 6 of the yellow blocks] she needs 6 more to make 12. Great, go ahead and write 6 in that box for us [the first missing addend box] so this is another way you could solve it. You could have 12 cubes and 6 cubes, put them next to each other, and you know that you need 6 more cubes to equal 12. Whoa. Thank you Cassie.

Penny was practicing *willing to struggle* as she pursued Cassie's direct modeling strategy of "joining to" to help her find the missing addend. Without Penny's specific follow up questions, Cassie's thinking would not have been as thoroughly explored. Though Penny could have asked Cassie what the "there's six more blocks" meant (talk turn 8), the discussion demonstrated Penny's *willingness to struggle.*

Changes in Penny's use of willing to struggle. The LPI appeared to change Penny's willingness to struggle to understand student thinking. The LPI had a slight effect on her lesson planning and a stronger effect on her instructional practice of *willing to struggle* in lessons two and three. Penny summed up her thoughts about going deeper with student thinking:

> If I felt like the kid had more to say and just needed to phrase it better or to ask a deeper question or to ask in a different way, then I went ahead and did that. For a kid to have it all up in their head is one thing, but to get it out in words is incredibly difficult, so asking them to do that is deeper, as opposed to me saying who got the answer, what's the answer? If they know it from memorization that's not really showing why they know it or showing the actual reason why they know it, other than recall, so I'm asking them to analyze and do some higher Bloom's

> taxonomy order of thinking by having them verbalize what they did step by step process of what they did.

Penny and research questions one and two. The first question was, "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI (lesson plan intervention)?" Penny integrated seven elements into her first lesson plan. Her lesson did not include word problems, so it makes sense that this number would be lower than her post-LPI lesson plans. Her most common element in her first lesson plan was *invites students to share* their strategies. After the LPI, Penny demonstrated a prominent increase in the number of CGI elements she integrated into her lesson plans. Penny integrated 10 elements into her second lesson plan and 11 in her third, in contrast to 7 elements in her first lesson plan. The most common elements in her second lesson plan were *invites students to reflect* on others' strategies and *invites multiple strategies.* Her second lesson plan was substantively different in purpose and activities than her first lesson plan. Penny's third lesson plan was similar to her second, with *invites students to reflect* and *invites multiple strategies* as common elements. It appeared that these increases were at least partly due to the LPI. When speaking about integrating CGI elements into her lesson plans, Penny stated,

> I like the elements that I used these past couple of days. The direction of the lessons, where they were going, might have been a little crazy because I was trying to craft them to be a certain way. But I hope even through that, that students felt like they were challenged. I feel like we can tell them the answers, and tell him how to do things, but I feel like these lessons ask them to bring more to the table, to be more participants than observers. And I think that's really important.

Research Question Two asked, "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" In this study Penny showed an increase in the number of CGI elements she utilized in her enacted lessons. Of the 18 identified CGI elements, Penny enacted 7 in her first lesson, 15 in her second and 9 in her third. Given this increase it was reasonable to assume that the LPI sessions were weighty in Penny's decision to increase the number of elements she enacted. Her use of less elements in her third enacted lesson than in her second was possibly due to her focus on equations as the primary representation of solution strategies. Less use of CGI elements in her third lesson plan might also be contributed to her solo writing of her third lesson plan. Like Jennie, it was notable that Penny was successful in utilizing many of the 18 elements as early as the second lesson (15) and that this continued into her third enacted lesson (9). Penny concluded that,

> I think I took a lot away from this experience. I really value what they [students] have to say. It's so important to challenge them to put that into words, and I've always known that, but . . . I don't have enough time to hear what everyone thinks and I don't have enough time to know if they think something and it's wrong and I have to fix it. So to not be afraid to have them share and be the participants and take ownership of their learning, because that's what you are really asking them to do. To hop on board and have them be fully participating with you, actively engaged. Yes, I think I have a greater appreciation for the importance of that.

Penny and research question three. Research Question Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" Penny's first lesson teaching practices included direct instruction and directed discussion. These practices often result in helping students remember or explore mathematical ideas, but do not always foster higher order thinking needed to analyze solution strategies or to connect strategies with other strategies. Because her lesson did not contain word problems, she used fewer CGI elements than after the LPI sessions. The only association between direct instruction and CGI practices was *intentional listening* by Penny and also *showing respect* for her students' mathematical thinking. Much of her instruction followed a modeling sequence, where students watched her pointing to numbers on the SMART Board while students spoke them out loud. Some of Penny's directed instruction included asking lower order questions such as, "How many are there? How many were in this part?" Some of her questioning was conceptual, but in a direct instruction format, such as, "If you had zero here [ones place] does that mean you have zero there [tens place]?"

Changes in Penny's teaching practices. After the LPI sessions, Penny demonstrated changes in her instructional practices. Because her second enacted lesson utilized word problems (her first lesson did not), the changes were predictable. Her second enacted lesson demonstrated direct instruction, directed discussion, and inquiry-guided discussion pedagogies. During her second lesson, Penny's use of direct instruction was minimal, only occurring during her warm up review of two-digit numbers. Her use of directed discussion was less frequent than in her first lesson and was associated with five CGI elements, three of them being *showing respect* for student thinking, *uses CGI problem types*, and *uses word problems*. Penny's use of inquiry-guided discussion in her second lesson was extensive and associated with many CGI elements. As Penny used inquiry-guided discussion pedagogy, she progressed through her four word problems. She was successful in *inviting students to share* their mathematical strategies with open-ended questions. She also successfully *broadcasted* students' responses to the class, opening other models of mathematical strategies for students to contemplate. Penny's use of inquiry-guided discussions allowed her to ask both general and specific follow up questions as well as *invite multiple strategies.*

An example of Penny's change in pedagogical practices emerged when one particular word problem proved to be difficult for the class to solve and highlighted Penny's practice of *willingness to struggle* to understand student thinking. In a missing addend problem, students were erroneously adding the sum and one addend, opening an opportunity for Penny to guide the discussion to the core of the mathematical content, that the relationship between the three numbers was not the same as just adding the two known numbers. This was an important distinction mathematically, and for the purposes of this study an important growth opportunity for Penny, as she worked to practice inquiry-guided discussions. A central aspect of the CGI framework is helping students orient themselves accurately around word problems, and Penny was engaged in doing this, using her knowledge of student thinking gleaned from the LPI sessions and her methods course.

In her third lesson, Penny demonstrated more direct instruction than in her second lesson. This might have been because Penny did not pose any word problems, though she had written them in her lesson plan. Instead, Penny posed equations ($3 + \_\_ = 10$) for class discussion. As Penny worked with the class to solve these missing addend equations, she practiced a balance of direct instruction and directed discussion. In this balance she integrated nine CGI elements, more than her first lesson, but less than her second lesson. It seemed that these two pedagogies worked somewhat well for Penny and yet left less room for some of the elements she used in her second lesson. It also appeared that less CGI elements co-occurred with less student-centered discussions. Elements that went missing were *using word problems* and *pursues student thinking to completion*. The omission of these two elements seemed to deter the third lesson's objectives for more in-depth exploration of student's initial ideas.

Penny's return to using symbolic representations (equations) was in some ways a surprise, but she did teach symbolic representations differently in her third lesson than in her first lesson. While using direct instruction and directed discussion, Penny seemed to value different strategies from students more than in her first lesson. In the interview after the third lesson, Penny expressed her reasoning for not enacting the word problems she included in her lesson plans. This will be discussed in Chapter Five.

Eleanor as a Case

Eleanor's participation in this study presented a unique opportunity to look at how a CGI framework-based LPI might affect her teaching practice during student teaching. Eleanor had a high level of participation in my previous mathematics methods course and also a high interest in CGI practices. Her recalled knowledge of CGI practices was commensurate with Jennie's and Penny's, but she asked different kinds of questions in class and appeared to welcome new concepts that the CGI framework brought to mathematics education. Eleanor was also unique from the other two participants in her collegiate accomplishments, life experiences, and mathematics background. A few years previous to this study, Eleanor had completed a bachelor' degree and had returned to college to complete her licensure in elementary education. Also unique from Jennie and Penny, Eleanor was not pursuing additional licensure in pre-primary education (covering ages 3 years to third grade). Eleanor was married with two children and was in her early thirties.

Eleanor often stated that she learned mathematics differently as a child and enjoyed finding patterns and relationships with numbers. “There was lots of crying over math problems at the kitchen table, a mom (and teacher) who hated it (my interpretation). This is probably where my indifference came. I remember doing homework with my dad’s friend and him making it understandable.” When asked how she felt about mathematics, Eleanor responded, “Honestly, I’ve felt indifferent. Not too excited about it, but since I’m not terrible at it, I’ve never minded it either. Once I get into a problem, I might actually enjoy it, like a puzzle to solve, but I would usually avoid it [math problems] if possible.”

Reflecting on her collegiate mathematics experiences, Eleanor stated, “In college [her first degree program] in the mathematics survey [course], math actually made sense and I wished I had a teacher who cared about math as much as Dr. Ironside (pseudonym) before. Unfortunately he was the first teacher I had who really showed a love for math and explained it in an understanding way.”

Her conflict of feelings about mathematics was real, but did not stop her from volunteering for this study or from seeing it all the way through. “With this new math group [in her placement classroom] it has been fun to watch them learn. They have a new look at math so far, so I am happy to keep that new and exciting.” Although this conflicting stance with mathematics was reported by all the PSETs to some degree, Eleanor kept an optimistic view about her ability to teach math, both during the study and throughout the length of her student teaching placement.

The following sections highlight Eleanor’s integration of four CGI elements into her lesson plans, LPI sessions, and enacted lessons as they occurred in an authentic learning environment. The four elements were: (a) *broadcasting,* (b) *invites multiple strategies,* (c) *invites reflections,* and (d) *pursues student thinking to completion.* Through Eleanor's use of these four elements, data emerged that helped me understand how she utilized CGI framework as a tool to help her elicit, interpret, and utilize student thinking to develop her first graders' number sense.

Eleanor's utilization of CGI framework elements.

Table 6

*Elements of CGI framework in **Eleanor's Lesson Plans.** Each row is the same CGI element. There were 18 CGI elements in this study. The three highlighted elements were the focus elements of analysis in Eleanor's lesson plans.*

Pre-intervention Lesson Plan (1)	Intervention Lesson Plan (2)	**Post-**intervention Lesson Plan (3)
Expects students to pursue solutions strategies	Expects students to pursue solutions strategies	Expects students to pursue solutions strategies
Uses word problems to build number sense	Uses word problems to build number sense	Uses word problems to build number sense
Broadcasts students' solutions	Broadcasts students' solutions	Broadcasts students' solutions
Invites students to share solution strategies	Invites students to share solution strategies	Invites students to share solution strategies
Invites multiple strategies	Invites multiple strategies	Invites multiple strategies
Uses CGI framework problem types	Uses CGI framework problem types	Uses CGI framework problem types
Expects students to pursue solution strategies	Expects students to pursue solution strategies	Expects students to pursue solution strategies
Pursues invented algorithms		
	Intentional listening to student thinking	
8 elements ↑	Presents problems without modeling solution	Presents problems without modeling solution
	Invites students to reflect on other students' thinking	Invites students to reflect on other students' thinking
	10 elements ↑	Flexible range of numbers
		10 elements ↑

Table 7

Elements of CGI framework in ***Eleanor's Enacted Lessons****. Each row is the same element. There were 18 CGI elements in this study. The four highlighted elements were the focus elements of analysis in Eleanor's work.*

First Lesson	Second Lesson	Third Lesson
Expects students to pursue solution strategies	Expects students to pursue solution strategies	Expects students to pursue solution strategies
Invites students to share solution strategies	Invites students to share solution strategies	Invites students to share solution strategies
Broadcasting	Broadcasting	Broadcasting
Builds on students' starting points	Builds on students' starting points	Builds on students' starting points
Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities	Uses students' intuitive problem solving abilities
Pursues student thinking to completion	Pursues student thinking to completion	
Children showing respect for others' thinking	Children showing respect for others' thinking	
7 Elements ↑	Uses CGI framework problem types	
	Invites students to reflect on other student's thinking	
	Uses word problems to build number sense	
	Teacher learns after listening	
	Invites multiple strategies	Invites multiple strategies
	Presents problems without modeling	Presents problems without modeling
	Willing to struggle to understand student thinking	Willing to struggle to understand student thinking
	Intentional listening to student thinking	Intentional listening to student thinking
	15 Elements ↑	9 Elements ↑

Eleanor's use of broadcasting. As a reminder, *broadcasting* is my descriptor for the CGI teaching practice of revealing student statements, questions, or reflections known to the whole group. When it occurred in a lesson, *broadcasting* could be applied to initial student responses, follow up questions, and student reflections of other students' thinking.

Broadcasting in lesson plan one and enacted lesson one. Eleanor's first lesson plan engaged students in solving two different, addition CGI framework problem types as well as learning about her "making a 10" invented algorithm. Six students volunteered to share their thinking during the lesson. Eleanor's lesson plan did not specifically call for Eleanor to *broadcast* (to the class) any specific dialog or action, but did make room for students to share strategies and answers. Eleanor's lesson plan also called for *broadcasting* her direct instruction for "making 10s."

In her first enacted lesson, Eleanor shared her own solution strategies for the class to ponder. Eleanor shared several students' truncated responses to her closed ended questions. Discussing the story problem, "14 children are playing games. 9 of them are girls. How many are boys," revealed:

1 Child:	[Wrote number 10 with 4 circles on the SMART Board]
2 Eleanor:	Okay so Aden made a 10 right away, and then he added 4 more [Eleanor pointing to his work on the board] so how does that show you how many boys there are? How many boys? What number should we be making? [Pointing to the 9]
3 Child:	9.
4 Eleanor:	We know that it's the number 9, right. So when you did this part [Eleanor pointing to the four circles next to the 10]. What were you thinking when you made that 10?
5 Child:	[Paused]
6 Eleanor:	Were you counting on?
7 Child:	Yeah.

8 Eleanor:	Yes. He was counting on. So if we were counting on there's a 10 so we're going to count on from 9 [pointing to the nine in the problem] 10, 11, 12, 13, 14. So how many boys are there?
9 Child:	5.
10 Eleanor:	That's why it might be confusing if you write the 10. Maybe you might want to start with 9 and then we make a circle with the 10.
11 Child:	[erasing the number 10]
12 Eleanor:	Okay there we go. Okay so do you want to write the 9?
13 Child:	[Writes the number 9]
14 Eleanor:	Perfect, so that means we need one more circle, don't we? Okay let's count it. He started with a 9 [pointing to the 9] 10, 11, 12, 13, 14. So Aden used counting on. So did you figure out the answer [asking the class] how many boys are there?
15 Child:	5.

Penny's use of directed instruction gave her first graders a limited opportunity to explore a missing addend problem. At two points in the conversation (talk turns 4 and 8) Penny *broadcast* a student's response to the problem. However, in talk turn 4, Penny asked mostly closed questions and thus limited the student's opportunity to explain her thinking.

Broadcasting in LPI sessions. The first session was similar to Jennie's and Penny's, with a review of CGI principles and elements. I explained *broadcasting* (not using the term) as, "Taking time to really listen, [to the student] 'I think this is what you're saying, you tell me, is this what you're saying, or are you saying something different?' So it's very student-centered."

In the second session, Eleanor and I analyzed videos of students solving problems, paying attention to how students' solution strategies coincided with problem types. We also analyzed how the instructor in the videos *broadcasted* students' thinking as a way of bringing out mathematical concepts for exploration. As we co-created the lesson plan, I explained what broadcasting could look like, emphasizing an intentionality for making students' statements (correct, incorrect, or partially correct) known to the rest of the group.

Broadcasting in lesson plan two and enacted lesson two. Eleanor's second lesson plan goals were to have students order numbers from 100 to 128 and to experience two different CGI problem types (join, change unknown and part part whole, whole unknown). Broadcasting was included in Eleanor's second lesson plan through: (a) student sharing initial strategy, (b) teacher clarifying their strategy, (c) inviting reflections/comments from other students (What do you think about how Susie solved the problem?), and (d) inviting other students to share different strategies. Her second lesson plan was a change from her first, giving learning space for students to encounter the mathematical strategies under consideration because she allowed for *broadcasting*. Eight students volunteered to share their thinking during the lesson.

In her second enacted lesson, Eleanor demonstrated *broadcasting* in eight of her one-to-one discussions in front of the class. One vignette went like this:

1 Eleanor:	Can you explain to the class how you did it?
2 Brenda:	I put 8 counters on one side [child had 8 counters in a circle and 7 counters in another circle and 8 + 7 = 15 at the top of the whiteboard].
3 Eleanor:	So you put 8 counters on one side. How did you do that? Did you put the counters right there? You circled it I see. And then what did you decide to do after that?
4 Brenda:	I can't remember.
5 Eleanor:	You can't remember, okay. Look on your board. Boys and girls, can you see what Brenda did over here? She put 8 together, right, and then she circled it. Then how did you decide what goes in the other box?
6 Brenda:	I counted on with the counter.
7 Eleanor:	You counted on with a counter. Okay show me what you did with the counters.
8 Brenda:	I counted by twos up until the last one.

9 Eleanor: Okay. So boys and girls, Brenda counted by twos when she was trying to get the counters together. She counted by twos up to 8, then she counted by twos up to 6 and added one.

Eleanor was successful *broadcasting* the discussion for the class to mull over. In talk turn 9 it was evident that Eleanor was intentional about keeping the rest of the class engaged in the strategy that Brenda was using. Eleanor's second enacted lesson was significantly different than her first with regard to her use of the element of *broadcasting*. Eight vignettes contained many instances of Eleanor *broadcasting* student thinking and students' reflections of other students' thinking. It was evident that the LPI had influenced Eleanor's practice of *broadcasting*.

Broadcasting in lesson plan three and enacted lesson three. Eleanor's third lesson plan called for three story problems to be posed for students. Two were join, start unknown (___ + 4 = 15) and one was a compare, compare quantity unknown (Irene has 12 cards, Helena has 7 cards more than Irene, how many cards does Helena have?). Three students volunteered to share their thinking during the lesson. Eleanor integrated *broadcasting* with students sharing their initial thinking, sharing multiple strategies and student-to-student sharing. Eleanor described student-to-student sharing as, "Have students' partners share how they got their answer, showing each other." Additionally, Eleanor amplified this student-to-student learning by having student pairs share their thinking with the whole class. She wrote that she would, "Call on students to share how their partner got their answer." Her *broadcasting* of these student-to-student sharings was new pedagogy for Eleanor and appeared to demonstrate an effect of the LPI on her lesson planning practice.

Eleanor's use of *broadcasting* emerged several times in her third enacted lesson. While solving a start unknown story problem (___+ 6 = 13), Eleanor was actively *broadcasting* Maise's thinking in front of a large group:

1 Eleanor:	Maise, how did you figure it out?
2 Maise:	I wrote a 6 and I circled it and then I counted up to 13 and I made circles and I counted the circles. I figured out it was 7 so I wrote the 7.
3 Eleanor:	Okay, can you hold that up please [child holding up worksheet]. Boys and girls, can you see what Maise did? She put a 6 down and then she drew 6 circles. So then Maise, you started at 6 and counted on from 6?
4 Maise:	6.
5 Eleanor:	Did you count up from 6 first, or did you count on from 6?
6 Maise:	I counted on from 6.
7 Eleanor:	Okay you started from 7. Show me how you did that. Just point to what you did. Do you see those circles that she drew? Can you count on for us, Maise? How did you do it?
8 Maise:	6 [pausing to move 6 counters to one side] 7, 8, 9, 10, 11, 12, 13. And then I counted these circles [counters separated from the 6] and that's 7.
9 Eleanor:	So she counted on with the circles first and then she counted them again to see how many circles she made.

Eleanor interpreted Maise's mathematical strategy and successfully *broadcasted* much of what Maise said for the class to consider. Eleanor was also cognizant of the two common strategies for solving this join, start unknown problem type and pursued Maise's thoughts with questions that specifically addressed these strategies.

Changes in Eleanor's broadcasting. Eleanor's lesson plans changed in their utilization of the CGI element of *broadcasting*. Both her second and third lesson plans saw greater *broadcasting* of student thinking through specific questioning. Her third lesson plan took *broadcasting* to another level by giving space for student-to-student discussions to be shared with the rest of the class. These instances lend credence to the

LPI having an effect on Eleanor's ability to integrate *broadcasting* into her lesson planning practices.

Eleanor's enacted lessons changed over time in her integration of *broadcasting* of student thinking. Her first lesson demonstrated limited use of *broadcasting* (mostly Eleanor's own thinking) while her second lesson integrated *broadcasting* in eight student-to-teacher discourses. This was a significant increase. By the third enacted lesson, Eleanor had elevated her use of *broadcasting* to also include student-to-student turn and talks being shared with the whole group. It seems likely that the LPI helped increase Eleanor's practice of *broadcasting* student mathematical thinking as the study progressed.

Eleanor's use of invites multiple strategies. As a reminder, the descriptor *invites multiple strategies* characterizes a teacher's act of inviting students to share a different solution (or perceived different solution) strategy(ies) than one shared previously. The comparison of strategies helps reveal mathematical patterns and opens doors for students to see their own and others' mathematical thinking in new ways.

Invites multiple strategies in lesson plan one and enacted lesson one. Eleanor was intentional in her first lesson plan to invite students to share *multiple strategies*. She mentioned this once in her lesson plan and scheduled five minutes for this to occur.

In Eleanor's first enacted lesson, she *invited multiple strategies* for a 14 = 9 + ___ story problem and the conversation sounded like this:

1 Eleanor:	You counted in your head and made a circle for them. You figured out that there were how many boys?
2 Child:	5.
3 Eleanor:	How many of you guys used a different way on your math board?

> [Looking at two students] Did you guys draw? How many of you used a different way? Let's give a hand to Shalisa and Mark. Is there a different way that we could do this? Is there a different way we could do this when we are drawing?

Though Eleanor *invited multiple strategies* only once in the lesson, she did follow up the invitation with specific questions that addressed the mathematics the child used as her strategy.

Invites multiple strategies in LPI sessions. Eleanor and I discussed *invites multiple strategies* twice in the LPI sessions. Both instances were in the context of *inviting students to reflect* on other students' strategies. In one of the instances I shared:

> So in the first one [students] share their strategy, you ask students, "How did you get your answer?" Then ask them to show us, make sure it's clear. Then you can make it clear to the class what they did. And then you ask, "Can you show me", as often as you can and then when that person has demonstrated [their solution strategy] you ask students, "What do you think about how Susie solved that problem?" Then students share that. Then you can ask, "Did anyone solve it another way?"

In this instance, I tried to enable Eleanor to utilize *invites multiple strategies* in conjunction with *invites reflections* to better situate Eleanor to interpret and compare the mathematics behind student strategies so she could help her students do the same.

Invites multiple strategies in lesson plan two and enacted lesson two. Eleanor's second, co-constructed lesson plan incorporated the *invites multiple strategies* element three times, right after each occurrence of *invites students to reflect* element. This was planned so that Eleanor would be situated to effectively elicit and interpret a substantial amount of students' mathematical thinking. As she guided these discussions, students would also be well situated to interpret and analyze the mathematics behind their classmates' thinking.

Invites multiple strategies occurred eight times in Eleanor's second enacted lesson. She used the phrase, "Did anyone do it differently?" in most of these instances. One of Eleanor's statements to the class depicts her thoughts:

> Brianna counted by twos when she was trying to get the counters together. She counted by twos up to 8, then she counted by twos up to 6 and added 1 [getting the correct missing addend of 7]. What do you guys think about what Brianna did? Did it make sense? . . . She came up with the same answer as Ariel but did you see how she did it differently? Is it okay for us to do things differently? [class says yeah] Absolutely! Who did it an even different way from Ariana and Brianna?

Eleanor's practice of *inviting multiple strategies* increased from her first lesson, where she *invited* just one time. She had clearly demonstrated a change in her teaching pedagogy. Eleanor's post-lesson conversation revealed her quandary about what constituted a "different" strategy and a perceived consequence of *inviting multiple strategie*s - feeling pressed to keep the lesson on pace.

> It was like they all wanted to share what they did. And I don't know what you would suggest for that. Because I still had other kids saying they did it differently. I don't know that they did it that much differently, but just a little bit different so they wanted to share, which I think is great, but I didn't want to stay too long on the one problem.

Invites multiple strategies in lesson plan three and enacted lesson three. Eleanor incorporated *invites multiple strategies* three times in her third lesson plan. Each occurrence is in the context of Eleanor asking multiple questions. For example, the first time Eleanor lists *invites multiple strategies* she also lists four other questions, "How many used the same way as before? Who tried a different way? Did you try a friend's way? What made you try?" Though it is somewhat problematic pedagogically to ask four questions at the same time, it is evident that Eleanor was purposeful about eliciting *multiple strategies* from her students. This was a change from her first and second lesson plans.

In her third enacted lesson, Eleanor *invited multiple strategies* many times. She also followed through most of these instances with at least two follow up questions. One occurrence of Eleanor *inviting multiple strategies* was particularly thorough - asking individuals if they used two different strategies themselves. This is an extension of this element and bears evidence to her growth in integrating this element.

1 Eleanor:	Raise your hand if you used the same strategy. [ten second pause] Okay, put your hands down. How many of you used two different strategies? [some students raising hands] I noticed a couple of you used two different strategies. Angel, can you explain that you used a different strategy?
2 Angel:	I did the 9, I did the double 9s.
3 Eleanor:	So what are the doubles, Angel?
4 Angel:	9 + 9 equals 18.
5 Eleanor:	9 + 9 equals 18. Did anyone else know that 9 + 9 equals 18? [raising her hand] The doubles. How many of you knew that was a double? Did you use that as a strategy or did you do something else? I want you to share with a partner if you used that strategy or did you do something else.

As Eleanor investigated Angel's thinking, Eleanor accomplished a number of tasks. She recognized, in real time, that several students used two strategies for this problem. She then asked a specific follow up of Angel's doubles strategy and *broadcast* Angel's explanation of it for the class to hear. Then she extended the learning opportunity through a partner sharing (turn and talk) activity the students were familiar with. Her teaching practice was very different from her first enacted lesson and still more sophisticated than what she demonstrated in her second lesson.

Changes in Eleanor's invites multiple strategies. From her first lesson plan to her third, changes emerged in Eleanor's integration of *invites multiple strategies*. She incorporated a little bit of the element in her first lesson plan and quite a bit more into her second lesson plan. Her third lesson plan integrated *invites multiple strategies* through a series of similar questions aimed at drawing out their strategies for the class to peruse. In the interview at the end of the study, Eleanor mentioned how she felt about inviting multiple strategies:

> I really liked it today when they had a couple kids share with each other because that was like even if they never use the strategy that was shown at least they can see it's there and then they can, I think, it just helps them solidify patterns in math because if they see or say, "I can do it this way, I can also do it this way" even if they never do it this way, they know that it works that way so they're not like stuck on it. So that they know there are other ways of doing things, so if they do ever get stuck in the way they've decided to do something, then they know there's another way to do it.

Eleanor's use of invites reflection. The descriptor *invites reflection* characterizes the CGI element that teachers can invite students to engage with and reflect on other students' mathematical thinking. *Invites reflection* illuminates important mathematical concepts that can be leveraged to build students' number sense.

Invites reflection in lesson plan one and enacted lesson one. The element *invites reflection* was not integrated into Eleanor's first lesson plan. She did, however, ask students to create story problems from equations (8 + __ = 14), which is a very difficult task for most primary grade students.

In Enacted Lesson One, Eleanor did not integrate *invites reflection* into her lesson, but she did ask students to compare two problems they worked on together. This was a difficult task and demonstrated Eleanor's practice of asking students to think about math equations in the context of story problems. Eleanor entered this study as a PSET with high expectations for her students and a willingness to pursue student thinking about equations and story problems.

Invites reflection in LPI sessions. In the first LPI session, I shared with Eleanor the practice of *inviting students to reflect on* other students' mathematical strategies. In the first session I mentioned this idea twice - both times in the context that word problems have varied levels of difficulty and require certain levels of mathematical sophistication to solve.

During the second LPI session, Eleanor integrated *invites reflection* into her second lesson plan. I shared with Eleanor, "It [*invites reflection*] is . . . about you helping them make clear their idea, and then expose that idea to other children and ask them what they think. I articulated the idea with, "Can you show me", and "What do you think about how Susie solved the problem?"

Invites reflection in lesson plan two and enacted lesson two. Eleanor integrated *invites reflection* six times into her second lesson plan, usually with the phrase, "What do you think about how Susie solved the problem?" This was a big change from her first lesson plan, which had no *invites reflections* in it. The five *invitations* were part of the LPI's effect on Eleanor's practice of incorporating student engagement with other students' mathematical thinking while problem solving. In the post-lesson interview, Eleanor shared that the LPI-generated second lesson plan was helpful:

> And the whole idea of getting behind their thinking and trying to expose that to the rest of the class, I have asked that before . . . "Who had a different way?" or "Who could do it a different way?" I've asked those kinds of questions before, but I haven't really given them the opportunity to explore each other's thinking versus knowing and that it's okay to do things different ways. I've done some of that but I haven't had them embrace other people's ways of doing things before.

In her second enacted lesson, Eleanor incorporated six instances of *inviting students to reflect* on a classmate's strategy. Three of these instances resulted in abbreviated follow through by Eleanor, one instance resulted in an "agree, disagree" question, and two instances resulted in Eleanor asking the class if a student's strategy was "good" or "made sense". The LPI seemed to have initiated Eleanor's practice of asking students about other students' strategies, even though her follow through tended to be brief. It is difficult pedagogy to orchestrate mathematical conversations that invites reflections from students while simultaneously addressing multiple solution strategies (Jacobs et al., 2010).

Invites reflection in lesson plan three and enacted lesson three. Eleanor included in her lesson plan to ask her students, "What do you think about how Susie solved the problem?" Though she included this question just one time, her lesson plan also included many *invitations to share* and open-ended questions intended to unpack multiple solution strategies.

In her third enacted lesson, Eleanor posed a missing addend word problem. Then she *invited reflection* from her students with, "What do you guys think about what McKenna did?" When one student replied, "She expected math", Eleanor did not pursue the conversation further. Eleanor's use of *invites reflection* decreased after her second lesson (six instances to one instance).

Changes in Eleanor's invites reflection. Eleanor's integration and use of *invites reflection* was episodic in this study. Her first lesson plan and enacted lesson did not use the element at all. Her second lesson plan included *invites reflection* six times and was enacted six times. Eleanor's third lesson plan integrated the element once and she enacted it once. She was consistent in its use, but did not pursue student reflections beyond brief responses.

Eleanor's use of pursues to completion. *Pursues to completion* is my descriptor for supporting students "to work all the way through the details of their strategies, with specific follow up questions drawing from what the student shared or did" (Carpenter et al., 2015, p. 149). *Pursues to completion* is a compilation of several CGI elements, including *invites students to share, building on student starting points, intentional listening to student thinking, invites reflection,* and asking general and specific follow up questions. The CGI element *pursues to completion* is analyzed in this study only through Eleanor. Her affinity for this element and her successful integration of it in large group settings warranted its analysis.

Pursues to completion in lesson plan one and enacted lesson one. Eleanor's first lesson plan did not address *pursues to completion* directly, but was inquiry-based and CGI framework-oriented, even before the LPI had begun. Her lesson plan called for students to solve two different CGI problem types related to missing addends. Eleanor's lesson plan also asked students to write a story problem based on a missing addend problem (8 + __ = 14), giving the lesson potential for substantive insights into children's thinking.

In Enacted Lesson One, Eleanor used an interactive form of direct instruction, where her speaking was occasionally interspersed with student comments and reflections. One such discussion was characteristic of her use of *pursues to completion*:

1 Eleanor: I'm giving you a problem [8 + ___ = 14 is on the SMART Board] Eight plus mystery equals 14. I want you to come up with a story problem to go with this. Let's come up with a story to go with the

problem. Who can give me the first part? Who wants to try doing a story problem?

2 Child:	There were eight cookies.
3 Eleanor:	Okay there were eight cookies [wrote "8 cookies" on the SMART Board] then, what did you say, Evie?
4 Evie:	Mom made more.
5 Eleanor:	Mom made more [wrote this on the SMART Board] Now there are how many cookies?
6 Child:	14.
7 Eleanor:	14 cookies. So what would our question be? [wrote "Now there are 14 cookies," on SMART Board]. What's our last question going to be? If there's eight cookies and Mom made more and now there are 14, what is our question at the end? [circled the missing addend box]. We need to figure out that last part, right? So if you know the answer, how would we say it? We would say, "How many cookies did Mom make? How many more cookies did Mom make?"

Eleanor asks her students to do a difficult task for first grade - translating a symbolic problem to a written one, a rigorous task (Lesh et al., 1987). In talk turn 3 Eleanor practiced *builds on student starting points* and in talk turn 4 catches Evie's thoughts on the missing addend, *broadcasting* it. Though Eleanor missed an opportunity for students to ask what Eleanor asked in talk turn 5, she still caught another child's response of "14" (talk turn 6) and *broadcasted* it. Together the conversation showed a partial version of *pursues to completion,* even before the LPI.

Pursues to completion in LPI sessions. The first LPI session reviewed the 18 elements of CGI framework, and I discussed with Eleanor some of the components of *pursues to completion:*

> They [CGI teachers] intentionally listen to student math thinking. You're going to pause and wait, we will plan that in this lesson and we'll see how you do that. And more than just wait time; I'll teach you how you can pursue their thinking. They demonstrate a willingness to struggle with identifying their [students'] thinking.

> Taking time to really listen, “I think this is what you're saying, you tell me, is this what you're saying, or are you saying something different?” So it's very student-centered.

The second LPI session gave Eleanor and I time to build her second lesson plan. After reviewing some CGI framework videos of teacher-to-student problem-solving sessions, we lesson planned to integrate several elements, including *pursues to completion*:

1 Researcher:	We are going to ask them to share their strategies. That's what the study is about. It is as much about you helping them make clear their idea, and then expose that idea to other children and ask them what they think. “Can you show me?” So when they are ready to share, then you will ask students, “Who would like to share?” So you will have one student share their strategy and then you ask yourself, “Did I specifically see what that child did?” So don't say, “He counted on his fingers”, you say, “Oh can you show me that?” If a child says, “I used my fingers or I drew 13 blocks. . .”
2 Eleanor:	Show me.
3 Researcher:	Can you show me?
4 Eleanor:	Can you say “show us”?
5 Researcher:	Yes, “Can you show us?” So you go back to the problem. Well, the bookshelf holds 15, and there are 8 there already, and they come up with 7, [you ask] how did you get your 7? The child would say, “I did . . . ” etc.

This vignette went full circle, from Eleanor inviting a student to share their strategy, to a student sharing, to Eleanor asking a general follow up question and finally a specific follow-up question. In a limited sense, *pursues to completion* was outlined in this instance.

Pursues to completion in lesson plan two and enacted lesson two. In her second lesson plan, Eleanor included most of the elements that make up *pursues to completion*:

invites students to share, building on starting points, intentional listening to student thinking, and asking general and specific follow up questions.

A thorough *pursues to completion* instance took place in Eleanor's second enacted lesson as the student responded to a part part whole, part unknown (missing addend) problem, "A book box holds 15 books. There are 8 books already in the book box. How many more books can fit in the book box?" The dialog went as follows:

1 Eleanor:	Okay, boys and girls, I want you to turn around and see what Evan did over there. [To Evan] Are those dots to represent books? So he drew the number 8 and then what did you do after that?
2 Evan:	I did the number 8, I did that [circled the number 8 and 2 more dots to make a 10], and then I did the number 10.
3 Eleanor:	Okay, he circled, is that what you did, circle the 10? 8 + 2 okay . . . and then were you able to see what you had left over after that?
4 Evan:	5.
5 Eleanor:	You have 5 left over and so then what does that make?
6 Evan:	15.
7 Eleanor:	Okay. So when you wrote the 8, how did you decide how many dots to draw?
8 Evan:	I did 7 dots
9 Eleanor:	You did 7. Were you counting on when you did that?
10 Evan:	Yes.
11 Eleanor:	Show me how you did that.
12 Evan:	[Erased his drawing].
13 Eleanor:	Boys and girls, you can try it while Evan is doing it. Can you count out loud, really loud so we can hear you?
14 Evan:	1, 2, 3, 4, 5, 6, 7 [Evan drew 7 dots and circled digit 8 plus 2 dots].
15 Eleanor:	You made a 10 and there's 5 left over, so you know there's 15 all together. What do you guys think about what Evan did?

In this vignette, Eleanor demonstrated *pursues to completion* through *inviting a student to share*, *building on student starting points* (talk turn 2), *intentional listening* (talk turns 3, 5, and 9), asking general follow up questions (talk turns 1 and 11), and asking specific follow up questions (talk turns 3, 5, 7, and 9). She also practiced the element of *broadcasting* in talk turns 3, 5, 9, and 15. Eleanor's instruction with Evan was a large contrast from her first lesson plan and gave evidence to the LPI's effect of helping Eleanor follow a student's thinking *through to completion* and building his number sense while *broadcasting* rich mathematical thinking to the whole class.

Pursues to completion in lesson plan three and enacted lesson three. Two practices related to *pursues to completion* were integrated into Eleanor's third lesson plan, including *builds on student starting points,* and asking general follow up questions. Eleanor recorded instances of *inviting students to share* with, "How did you get your answer? Show us."

Eleanor's third enacted lesson revealed several instances of *pursuing students' thinking to completion*. The most thorough instance focused on the word problem, "Mrs. Nebson had some books. She bought 6 more Elephant and Piggie books at the school book fair. Now she has 13 Elephant and Piggie books. How many did she start with?" The conversation is examined for *pursues to completion*:

1 Eleanor:	Mindy, how did you figure it out?
2 Mindy:	I wrote a 6 and I circled it and then I counted up to 13 and I made circles and I counted the circles. I figured out it was 7 so I wrote the 7.
3 Eleanor:	Okay, can you hold that up, please [child holding up worksheet] Boys and girls, can you see what Mindy did? She put a 6 down and then she drew 6 circles. So then Mindy you started at 6 and counted on from 6?
4 Mindy:	6.
5 Eleanor:	Did you count up from 6 first, or did you count on from 6?
6 Mindy:	I counted on from 6.
7 Eleanor:	Okay you started from 7. Show me how you did that. Just point to what you did. Do you see those circles that she drew? Can you count on for us Mindy? How did you do it?

8 Mindy: 6 [paused to move 6 counters to one side] 7, 8, 9, 10, 11, 12, 13. And then I counted these circles [counters separated from the 6] and that's 7.

9 Eleanor: So she counted on with the circles first and then she counted them again to see how many circles she made.

All the components of *pursues to completion* were present in this vignette. Eleanor demonstrated skill in eliciting Mindy's original thoughts (talk turns 1 and 3), interpreting Mindy's strategy by asking specific questions (talk turns 3, 5, and 7), clarifying important mathematical moves (talk turns 3, 5, and 9), and connecting Mindy's visual translation (circles) to the mathematics at hand (talk turn 9).

In her post-lesson interview, Eleanor shared her thoughts about the difficulty of pursuing one student's thinking:

1 Eleanor: I don't know if it is just because he's [student] not used to this whole idea [of showing your work] but then he told me it [the answer] was 19, which was correct, but there was not a 19 anywhere here on his worksheet. He had a 4 in the circle at first, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, again, so he never got up to, he never got up to 19, ever.

2 Researcher: Yes.

3 Eleanor: But he knew the answer when I asked him.

4 Researcher: Yes, I think with more time to unpack that [with him] we could have figured out what he was trying to do.

5 Eleanor: After like an hour [laughter] because I was talking for quite a while and I still couldn't figure out what he was doing.

Changes in Eleanor's pursues to completion. Evidence of Eleanor's *pursues to completion* was demonstrated in her patience (talk turn 5) while trying to interpret student thinking. By her third enacted lesson, Eleanor demonstrated greater skill in planning for and *pursuing student thinking to completion* during problem solving discussions. This was evidenced in several instances of Eleanor planning for and carrying out specific and general follow up questions and not making premature assessments of their mathematical thinking. It seemed probable that the LPI sessions equipped Eleanor to initiate, tend, and *pursue to completion* problem solving sessions with her first grade students.

Eleanor and research questions one and two. Research Question One asked, "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI (lesson plan intervention)?" By the completion of the study, Eleanor demonstrated a noticeable increase in the number of CGI elements she integrated into her lesson plans. Eleanor integrated 10 elements into her second and third lesson plans, in contrast to 8 elements in her first lesson plan. It appeared that these increases were at least partly due to the LPI. When sharing about how the LPI shaped her lessons, Eleanor shared,

> It was good. It was hard, when I went to write it [Lesson Plan Three]. How do I write a CGI lesson plan that's different from the one that I just did [second lesson plan]? And then I thought maybe it doesn't need to be that different. I picked different problems and I picked different types of problems from the sheet that you gave me [CGI problem types] so you saw the ones that I picked, different ones that were just a tiny bit harder than last time.

Research Question Two asked, "What elements of CGI do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" In this study, Eleanor showed an increase in the number of CGI elements she utilized in her enacted lessons. Of the 18 identified CGI elements, Eleanor enacted 7 in her first lesson, 15 in her second, and 9 in her third. These numbers were, coincidentally, identical to Penny's enacted elements. It was evidentiary to surmise that the LPI sessions had an effect on Eleanor's decision to increase the number of elements she enacted. The decrease in integrated elements from the second to the third lesson was possibly due to the third lesson's emphasis on partner sharing, and it was also a shorter lesson. Less use of CGI elements in her third lesson plan might also be contributed to her solo writing of her third lesson plan. Like both Jennie and Penny, it was encouraging that Eleanor was successful in integrating many of the 18 elements as early as the second lesson (15) and that this continued into her third enacted lesson (9). Eleanor concluded her thoughts about the LPI's emphasis on CGI,

> I think the flexibility of it, like letting them do that however they want helps you to explore their thinking. If you're requiring them to do it a certain way then you're not really able to see what they are thinking. Then they're going to do it just to do it. Not necessarily to figure out the answer. I really liked it today when they had a couple kids share with each other because that was like even if they never use the strategy that was shown at least they can see it's there.

Eleanor and research question three. Research Question Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" Eleanor's teaching practices situated her as a learning coach, with a relational style that encouraged students to speak spontaneously. She did not require students to raise their hands when responding and used extensive wait times to ensure students had sufficient time to reflect on the statement or question at hand. During all three lessons Eleanor welcomed one-to-one discussions while in large group settings.

In her first lesson, Eleanor disclosed her "making a 10" strategy as a way of solving word problems. She used direct instruction in the beginning of the lesson and moved to more directed discussion later in the lesson. In her direct instruction she utilized the CGI elements of *broadcasting* and *building on student starting points. Pursued student thinking to completion* was somewhat associated with her direct instruction. In Eleanor's directed instruction, she utilized the elements of *invited students* *to share* their strategies, *built on student starting points, broadcasting,* and *pursued student thinking to completion*. These associations indicated that some CGI elements fit into Eleanor's practice of directed instruction.

In her second lesson, some associations occurred between CGI elements and different teaching strategies. In this lesson, Eleanor utilized the same teaching techniques as her first lesson, but added inquiry-guided discussion. The majority of her second lesson was inquiry-guided discussion, where Eleanor asked students significantly more questions, both general and specific, than in her first lesson. CGI elements associated with her inquiry-guided discussion were many, including: (a) *pursuing student thinking to completion*, (b) *inviting students to reflect on other students' thinking,* (c) Eleanor *learning after listening to students*, (d) *inviting multiple strategies,* and (e) *willing to struggle to understand student thinking.* A major difference in her second lesson was that Eleanor did not interject her own teaching strategies into the lesson while using these elements. Students engaged in discourses, and mathematical thinking was elicited and displayed. This was an important distinction in her instruction and suggests that integrating specific CGI elements into inquiry-guided instruction could help PSETs elicit and utilize student thinking for sense making.

In her third lesson, as in her second, Eleanor demonstrated some changes in her instructional practices. With these changes came some associations between CGI elements and certain instructional models. One of these changes was that Eleanor did not focus on any specific solution strategy in this lesson, as she did in her first (which focused on the "making a 10" strategy). Instead, Eleanor practiced inquiry-guided instruction to ask general and specific follow-up questions from their initial responses.

Elements associated with the inquiry-guided sections of her lesson were *inviting multiple strategies*, *presents problems without modeling*, and *willing to struggle to understand student thinking*. These occurred when she asked students if they had different strategies they wanted to share and when Eleanor paused many times to listen to students' responses to her initial and follow-up questions. A second change in Eleanor's instructional practice occurred when Eleanor used *flexible number sets*, where students experienced a similar word problem, but with different numbers. Adding this element through her use of inquiry-guided discussions allowed Eleanor to elicit thinking about how strategies can be used with different numbers, a misconception common with younger children. A previously unobserved instructional practice, student-peer feedback, emerged when Eleanor utilized a form of the CGI element *inviting students to reflect on other students' thinking*. Calling it "partner sharing", Eleanor asked students to explain their solution strategies to a partner. In a sense, her partner sharing/student-peer feedback was a form of *inviting students to reflect on each other's thinking*. Both instructional strategies seek student-to-student engagement about word problem strategies.

Changes in Eleanor's teaching practices. Eleanor's stance as a relation-oriented teacher did not change during the study, working with students in the manner of an instructional coach. However, as she added teaching strategies in her second and third lessons, Eleanor incorporated more CGI elements into her instruction. When she utilized directed-discussion instruction, several CGI elements emerged, including *pursuing thinking to completion, inviting multiple strategies,* and *willingness to struggle to understand student thinking*. When Eleanor practiced inquiry-guided instruction, even more elements emerged, including *inviting students to reflect on other students' thinking* and *pursuing student thinking to completion.* As CGI's framework values discussion-based lessons and student-generated thinking, it seemed reasonable that it would blend well with both directed discussion and inquiry-guided methodologies.

Eleanor's second and third lesson use of inquiry-guided methodology also included *presenting problems without modeling* and *intentional listening to student thinking.* This association of elements with inquiry-guided instruction demonstrated Eleanor's commitment to elicit student thinking through discussion, questioning, and intentional listening to student thinking. Eleanor stated that:

> I guess my main point is to say that if you're teaching a specific way to do something then you don't really have any idea if that's how the kid is thinking, versus if they have the freedom to do whatever they want then you might see something. I'm not saying you will. There are still those kids that don't want to show you their work [but] at least you have a better chance of seeing something than you did if you were just making them do it a certain way or teaching them a certain algorithm or something.

Summary of Cases for Questions One and Two

Table 8

Presence or non-presence of six CGI elements in PSETs' lesson plan interventions, lesson plans, and enacted lessons. Elements not present are highlighted.

Group	Broadcasts student thinking (Br)	Invites student reflection (ISR)	Invites multiple strategies (IMS)	Various fourth elements
Jennie				Builds on student thinking (BST)
Lesson plan 1	Br	not present	IMS	BST
Enacted lesson 1	Br	not present	IMS	BST
Lesson plan intervention	Br	ISR	IMS	BST
Lesson plan 2	Br	ISR	IMS	BST
Enacted lesson 2	Br	ISR	IMS	BST
Lesson plan 3	Br	ISR	IMS	BST
Enacted lesson 3	Br	ISR	IMS	BST
Penny				Willing to struggle (WTS)
Lesson plan 1	Br	not present	not present	not present
Enacted lesson 1	Br	not present	not present	WTS
Lesson plan intervention	Br	ISR	IMS	WTS
Lesson plan 2	not present	ISR	IMS	WTS
Enacted lesson 2	Br	ISR	IMS	WTS
Lesson plan 3	Br	ISR	IMS	WTS
Enacted lesson 3	Br	ISR	IMS	WTS
Eleanor				Pursues to completion (PtC)
Lesson plan 1	Br	not present	IMS	not present
Enacted lesson 1	Br	not present	IMS	PtC
Lesson plan intervention	Br	ISR	IMS	PtC
Lesson plan 2	Br	ISR	IMS	PtC
Enacted lesson 2	Br	ISR	IMS	PtC
Lesson plan 3	Br	ISR	IMS	PtC
Enacted lesson 3	Br	ISR	IMS	not present

Jennie's, Penny's, and Eleanor's integration of six CGI elements varied as they sought to elicit, interpret, and utilize student thinking for sense making. Of the four .elements under analysis in Jennie's work, she integrated a new element (*invites students reflections)* in her second and third lesson plans and enacted lessons, suggesting the LPI made incorporation of this element (and others) a realistic practice for Jennie. Of the four elements under analysis in Penny's work, Penny integrated two new elements (*invites student reflection* and *invites multiple strategies*) into her work after the LPI, suggesting the LPI influenced Penny's use of CGI elements. Similarly, Eleanor integrated a new element (of the four analyzed) into her work after the LPI sessions, giving credence to the LPI's effect as an intercessory tool in Eleanor's efforts to integrate CGI elements for eliciting student thinking.

Summary of Cases for Question Three

Research Questions Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" All three PSETs demonstrated changes in their teaching practices during the course of this study - from direct instruction models in their first lessons, to more discussion-based and inquiry-focused models in their second and third lessons. With these changes came an association of some CGI elements with certain instructional methods. When Jennie, Penny, and Eleanor began to practice directed discussions, they utilized *inviting students to share strategies*, *intentional listening to student thinking*, and *inviting multiple strategies*. As the PSETs practiced the instructional method of inquiry-guided discussions, two CGI elements co-occurred: *willing to struggle to understand student thinking* and *invites students to reflect on other students' thinking.* These elements have similarities to both directed discussions and inquiry-guided discussion methodologies.

As Jennie, Penny, and Eleanor progressed through the study, they demonstrated remarkable pliability in their instructional practices. Visible through their efforts, they clearly valued understanding student strategies and having discussions based on them.

To varying degrees, they each were successful in having conversations that elicited more mathematical thinking than in their first lesson plans and lessons. Penny shared her thoughts on student thinking:

> I really value what they have to say. It's so important to challenge them to put that into words, and I've always known that, but I think I've been trying to take on so much, actually, teaching math . . . to not be afraid to have them share and be the participants and take ownership of their learning, because that's what you are really, what you're asking them to do.

Chapter Four articulated the findings of the study - the presence and utilization of four CGI elements by PSETs to elicit and utilize student thinking. All three PSETs showed differing uses of the elements of *broadcasting*, *invites students to reflect*, and *invites multiple strategies*.

Chapter Five: Discussion

This study explored a lesson plan intervention (LPI) for preservice elementary teachers (PSETs) during their student teaching placements. The purpose of the LPI was to equip PSETs to create lessons plans that, when enacted, would build number sense in their first grade students. Building number sense in children is an important outcome of quality elementary mathematics instruction and is a vital component to elementary students' academic success (National Council of Teachers of Mathematics, 2000). The importance of children's number sense, also called sense making, is evidenced in the mathematical practices of: reasoning abstractly, constructing viable arguments, modeling, and critiquing the reasoning of others (National Governors' Association Center for Best Practices and the Council of Chief State School Officers, 2014). Sense making is demonstrated in several ways. One of these is when students become flexible problem solvers, manipulating mathematical strategies through discussion and exploration. Sense making occurs when students explore conceptual relationships in context. Sense making becomes manifest when students justify and explain their mathematical reasoning (National Council of Teachers of Mathematics, 2014).

To incorporate student sense making in the PSETs' first grade mathematics lesson plans, the study sought a research-based framework that would be appropriate for this purpose. Carpenter et al.'s. (2015) framework, called Cognitively Guided Instruction (CGI), was a good fit for building sense making into lesson plans. CGI framework states that one way to build mathematical sense making in students is to understand and leverage students' own thinking while engaged in problem solving activities. To leverage student mathematical thinking meant that PSETs first had to have a way to interpret students' common ways of thinking as they worked on word problems. CGI framework was uniquely suitable for interpreting student mathematical thinking, as CGI practices help teachers understand that children go through a progression in their problem solving strategies and that this progression occurs naturally, but can be enhanced by guiding them through the concepts underlying students' mathematical strategies (Carpenter et al., 2015). Using CGI tenets, PSETs could then accurately interpret students' thinking and solution strategies. Once student thinking was interpreted for its mathematical concepts, then PSETs could use CGI's framework of children's problem solving strategies to guide mathematics discussions in ways that students would build number sense from their own and others' thinking.

The LPI used CGI framework to support PSETs' efforts to elicit student thinking to build student number sense. The PSETs had experienced CGI tenets three semesters prior to the study in a mathematics methods course taught by the researcher. Instruction in CGI elements was a significant part of the course, where PSETs were introduced to the elements of CGI framework and were given practice in analyzing children's thinking as they solved word problems.

An issue with integrating CGI elements into PSETs' lesson plans was the three semester time gap that elapsed since the PSETs had explored CGI principles in the methods course. An LPI could help alleviate PSETs' loss of CGI framework thinking that might naturally occur in such a gap, solving one of the problems associated with the merging of methods course material into PSETs' field experiences (Valencia et al., 2009). The LPI would act as a bridge to refresh PSETs' CGI framework thinking and increase their skill in eliciting, interpreting, and supporting students' thinking for number sense.

With the LPI's purpose and design in place, the research questions were formulated to determine what effect, if any, the LPI had on the PSETs' efforts to integrate CGI principles into their lesson plans and enacted lessons. As a reminder, the research questions were: (1) "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?" (2) "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" (3) "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?"

Summary of Procedures

To answer the research questions, a qualitative design was formulated that allowed for data collection with student teachers in-situ, as they were writing lessons and enacting them in their first grade classrooms. The study began with two pre-assessments given to potential PSETs. One was a CGI framework self-assessment to gather baseline characteristics of PSETs' CGI knowledge. The other was a mathematics self-assessment to gather data about PSETs' characteristics, feelings, and experiences about mathematics. The next part of the study asked PSETs to create a lesson plan in early number that involved word problems for problem solving. PSETs then enacted this first lesson plan, and observations took place to record the presence or absence of CGI elements as well as teaching practices. Following this first lesson, a conversation and semi-structured interview was conducted. Next, two LPI sessions took place between the researcher and each PSET, the first session to review elements of CGI framework and the second session to integrate CGI elements into an early number word problem lesson. Following the LPI sessions, each PSET enacted the lesson plan, and observations took place to record elements of CGI framework and teaching practices. Another set of conversations and interviews with each PSET ensued the same day. Finally, each PSET was asked to write a third lesson plan for early number problem solving. Then the PSETs enacted the lesson with their first graders and observations were made to record elements of CGI framework and teaching practices. The last steps in the study were another post-lesson conversation and an end-of-day interview with each PSET.

To interpret my findings, I reviewed literature related to six components of the study that were delineated in Chapter Two: (a) teaching for mathematical sense making, (b) sense making via student thinking, (c) undergraduate teacher preparation and student thinking, (d) connecting methods course content with student teaching, (e) leveraging student thinking through CGI practices, and (f) a lesson plan intervention to explore student thinking. To answer the three research questions, case by case analyses help the reader get a holistic sense of each PSET's experiences in the LPI.

Discussion of PSETs and Research Questions

The following discussions highlight the experiences of the PSETs as they relate to the research questions. Analysis of each PSET's successes in the LPI are also discussed, with relationships to relevant literature. The intent is to tie PSETs' experiences with issues in teacher education and relevant literature.

Jennie and the LPI

Jennie's journey through the LPI demonstrated her practices in using CGI elements in her lesson planning and instruction to elicit her first graders' mathematical thinking. The LPI was successful in equipping Jennie to write lesson plans that integrated CGI elements into her word problem lessons. Changes in her lesson planning and enacted lessons were substantive and are discussed for each research question.

Research question one. Research Question One asked, “What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?” Analyses of Jennie’s lesson plans revealed that after the LPI she demonstrated an increase in the presence of CGI elements in her lesson plans. This change was Jennie’s increased use of the elements of *listening* to student thinking, *inviting students to reflect* on other students’ thinking, and *refraining from modelin*g strategies to her students. These are all student-centered elements effective for eliciting student thinking and were not present in her pre-LPI lesson plan.

The first LPI session was associated with Jennie’s increased incorporation of the above elements into her lesson plans. At several points in the first LPI session, Jennie voiced her assumption that most of the solutions strategies students utilize must come from a teacher. I responded that CGI framework research showed that students are not blank slates and have intuitive capacities for problem solving that should be elicited and leveraged as they are engaged in problem solving activities. The LPI session gave room for Jennie’s misconception to be exposed and analyzed and to be compared to CGI theory about students’ mathematical thinking. It may be a significant outcome of the first LPI session that it gave Jennie a safe learning space to wrestle with an important aspect of how children think about mathematics, leading to her increased understanding of their thinking. This compares to Rayner’s (2015) PSET intervention which revealed that video analysis and skills-based instruction could be used to improve PSETs’ collection of evidence of student learning and analyze it to propose different strategies to improve their lessons.

Later in the first LPI session, while viewing a video of a problem solving session, Jennie mentioned that a student's use of partial sums (an incremental algorithm) was unique and "would really help for place value". Jennie made this important conceptual connection in the conversational space afforded by the LPI session. The first LPI session appeared to have helped Jennie increase her understanding of student mathematical thinking and connect it to the important mathematics concept of place value.

In the first LPI session, Jennie also connected students' solution strategies (in the video vignettes) with the CGI framework solution strategy chart. She later used this knowledge as she wrote her second lesson plan, looking for problems with multiple solution strategies. This was another perceived benefit of the LPI, that Jennie moved beyond single solution strategies as sufficient for a student learning outcome.

Prior to the LPI, Jennie's lesson plan demonstrated her belief that mathematical strategies should be modeled by teachers. Her inclusion of math mountains, fact triangles, and equations in her first lesson plan was evidence of this. During the second session of the LPI, Jennie asked if these models were too concrete and if I thought she should model strategies for the students. I responded that instead of demonstrating strategies to her students, she should plan to let her students share their thinking and see what solution strategies emerge. These discussions about CGI framework's vision of the role of the teacher during problem solving sessions seemed to have some effect on Jennie's second and third lesson plans, which placed greater emphasis on eliciting student thinking. Through the discussions Jennie and I had in the second LPI session, I was able to impart a key CGI framework concept - that students have intuitive problem-solving capacities and their strategies can be leveraged for sense making if we elicit and utilize them in our instruction. This changes the role of the teacher from modeler of solution strategies to elicitor of student strategies. The conversational spaces of the second LPI session opened the door for this important concept to be reviewed and integrated into her lesson plan.

When asked about lesson planning, Jennie stated she needed to be prepared for lessons and felt strongly that thorough lesson plans were key to her leading good mathematics lessons. In the second LPI session, as Jennie and I were co-creating the second lesson plan, she jumped into the effort with enthusiasm and a genuine interest in using CGI framework to understand her first graders' mathematical thinking. Jennie was able to use the LPI's CGI framework training to connect a visual model (math mountains) with the CGI framework solution strategy of counting on. This demonstrated her connection with the differences between representations and the actions that occur in the four operations of math. Jennie seemed to understand how solution strategies can be demonstrated through various models and that the mathematical concepts behind a model are what CGI framework focuses on as teachers work through problems with students. This is similar to Osana and Royea's (2011) research with PSETs that used an intervention to improve PSETs knowledge of fraction concept understanding. Their study revealed that PSETs can struggle with evaluating the cognitive difficulty of elementary mathematics problems because of inadequate mathematics content knowledge and that their misunderstandings can be based on superficial, non-mathematical ideas. Their conclusions relate to this study in that Jennie's conception of "math mountains" may have changed during the study. Her first lesson seemed to reveal that she may have valued math mountains for their visual form rather than their potential for revealing the relationships between numbers depending on the operation involved.

Jennie stated that the co-constructed process of writing the second lesson plan helped her ask initial exploratory questions of students. Literature on PSETs' design of lesson plans reveals that PSETs' content knowledge in their teacher preparation program is related to their performance on writing lesson plans two to three years after they graduated (Morris & Hiebert, 2017). This relates to this study as the time gap between Jennie learning to write CGI framework-based lesson plans in her methods course and her first lesson plan for this study was one and a half years. Jennie's content knowledge about CGI in the methods course was related to her success in writing her second and third CGI-based lesson plans in the LPI study. The LPI seemed to help Jennie recall and utilize her CGI framework knowledge when creating CGI practice-based lesson plans.

Jennie seemed to be able to recall salient points of CGI elements during the first LPI session, although sometimes in a "protocol" (Jennie's term) mindset. In hindsight, I did not do enough to clarify this misconception - that CGI framework was not a set of procedures, but a framework of knowledge about how students solve word problems. Jennie's perception of CGI framework as a procedure was articulated in the LPI, but the misconception was not overtly addressed by me. Procedures can be associated with procedural minded thinking with PSETs (Coffey, 2004; Soto-Johnson, Liams, Oberg, Boschmans, & Hoffmeister, 2010).

I speculate that the conversation and interview after Jennie's second enacted lesson had some effect on her third lesson plan. This was a positive result and showed that Jennie was able to use these discussions to further her understanding of what a CGI-based lesson could accomplish. Similarities in the questions she asked and the order she asked them serve as evidence. Though the intent of the post-lesson conversation and interview was not tutorial in nature, the discussions were reflective in nature and the follow up questions I asked her could have been viewed as instructional or as advice from a mentor (Barnett, 1995). Looking at the relational aspects of the LPI, it seemed that the stance between Jennie and I could have been more complex for her than I imagined. Did Jennie perceive me as her mentor, professor, or researcher for this study? Was I still her professor? Though she volunteered for the study, she was still one of my advisees at the college, potentially adding another layer to how she envisioned her participation in the research. In this situation, Jennie could be seen as being situated in a potentially conflicting setting or a beneficial one (Zanting et al., 2001). The LPI appeared to blend these positionalities for Jennie, who seemed comfortable in being a study participant and non-supervised student teacher. I speculate this occurred because of the discourse-based nature of the LPI sessions, the week-long time together in her placement classroom, the unstructured post-lesson conversations, and our previous work together in four methods courses. Jennie's flexible personality and inquisitive mindset rightly account for her successes in the research and her student teaching responsibilities.

Jennie's third lesson plan included a "making a 10" strategy (a partial sums strategy), which was modeled by Jennie's cooperating teacher in the classroom. Jennie's emphasis of the "making a 10" strategy could be seen as pedagogy she found to be valued by her cooperating teacher, pedagogy Jennie found to be mathematically efficient, or pedagogy Jennie valued as a partial sums strategy. The LPI did not address cooperating teachers' effect(s) on PSETs' use of CGI framework in lesson planning. Jennie's cooperating teacher's use of partial sums strategy is similar to CGI framework's idea of invented algorithms, possibly adding to Jennie's use of CGI elements in her third lesson plan.

Perceiving herself as a "not very good math student" but an active seeker of more knowledge, others may have positioned Jennie to successfully notice student mathematical thinking (Amador, 2015; Jacobs et al., 2010), since she was open to how others (including her students) looked at mathematics problems. Her perception of herself as having weaknesses in mathematical content may have helped Jennie to identify with students who also had mathematics struggles. In the LPI sessions, Jennie stated that her students were the "low group" in that month's assessments. Although this idea is problematic, and was an area of conflict for Jennie, she believed her students needed more engagement in the concepts of problem solving and that they were capable learners. The LPI discussion opened the door for Jennie's deficit thinking to be explored and rebutted.

Research question two. The second research question asked, "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" The analyses of Jennie's instruction revealed a substantive increase in enacted CGI elements after the LPI. New CGI elements that emerged in Jennie's instruction included *intentional listening* to student thinking, *invites students to reflect* on other students' thinking, and *willing to struggle* to understand student thinking. Surprisingly, more elements emerged from her enacted lessons than what she wrote in her lesson plans. I would speculate that Jennie's enacting more elements than she included in her lesson plans was due in part to the LPI helping her better understand CGI framework's emphasis on asking follow up questions specific to students' responses, resulting in more mathematics content and strategies being shared and thus more opportunities for CGI elements to emerge. It was not anticipated that the LPI would be associated with more enacted elements than planned elements. It is interesting that more CGI elements emerged during Jennie's instruction than included in her lesson plan. The increase may have been Jennie's increasing comfort level with CGI material that occurred over the week she was involved in the study. This would stand in contrast to research that preservice teachers often do not implement newly acquired knowledge or beliefs right away (Brown & Borko, 1992). It is possible that the LPI performed its function by helping Jennie recall CGI framework from her methods course.

Of the elements she integrated, the biggest change from lesson one was *inviting students to reflect* on other students' thinking. Jennie did not practice this element in her first lesson. This pedagogical skill was reviewed in the LPI and involved the difficult skill of interpreting students' thinking during in-the-moment time frames, a difficult skill for teachers, especially new teachers (Edwards & Protheroe, 2003). Though Jennie did not always follow through with students' reflections on other students' initial responses, she was consistent in asking students what they thought about the initial sharer's thinking. Interpreting the mathematical ideas behind students' shared strategies can be learned over time (Barnhardt & van Es, 2015; Carpenter et al., 2015; Jacobs et al., 2010), but the brevity of the LPI would limit this skill.

It appeared that the LPI may have better situated Jennie to notice and interpret the mathematics behind students' strategies (Jacobs et al., 2010). Her follow up questions of students' initial responses were more conceptual after the LPI took place. As the LPI sessions revealed the connections between similar solution strategies, Jennie may have been adding to her conceptual knowledge about early number. As she shifted away from broadcasting strategies that were procedurally based to more conceptually based, she also may have returned to the more familiar pedagogy of direct instruction as a way of finding the familiar in the midst of the newness of CGI instruction (Brown & Borko, 1992).

It is possible that the LPI was beneficial for Jennie because of her openness to instruction in lesson planning, but also because of her apparent comfort with thinking conceptually. Jennie's comfort with conceptual thinking was evident in the conversations we had after her second lesson. In these conversations Jennie expressed her excitement about students exploring different "processes and methods that we can do to get a certain answer". Jennie's comfort with conceptual thinking was also evident as she discussed how students made connections between addition and subtraction, even though she did not teach these connections. From these discussions, and others, it seems reasonable that Jennie's comfort with conceptual thinking afforded growth in her conceptual understanding of how solution strategies are more than different procedures. Jennie appeared to be exploring how different solution strategies reveal student mathematical sophistication in problem solving, a key foundation for the CGI framework. This seemed to be a contributing factor in the LPI's effectiveness at helping Jennie incorporate CGI elements into her instruction. It seems logical that this symmetry would benefit PSETs with a bent for conceptually based instruction. It also seems reasonable that a more procedurally based PSET could benefit from the CGI framework-based LPI, given an openness to learning new pedagogies. Analysis of student thinking to explore mathematics concepts and Jennie's disposition for conceptual thinking over procedural thinking made her a good candidate for an LPI. This has implications for other LPIs that would integrate CGI framework practices.

Research question three. Research Question Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" After the LPI, Jennie's practices were more student centered and mathematics concept oriented. This was evident in Jennie's practice of repeating students' statements rather than reciting procedural solutions as she predominantly did in her first lesson. After the LPI sessions, Jennie asked more follow up questions, eliciting more mathematical ideas from her first graders. Asking specific follow up questions takes experience and is a difficult practice for teachers to learn (Franke et al., 2009). Subsumed under the more general practice of orchestrating productive mathematics discussions, asking specific follow up questions is also a difficult task for most teachers, requiring experience and time to be successful (Stein et al., 2008). Her second lesson integrated directed discussion and inquiry-guided discussion to elicit student thinking. Her third lesson elicited many student strategies aimed at finding multiple addends to single sums (14 and 15), with Jennie having to interpret students' strategies and deciding how to leverage these for sense making. For example, Jennie sometimes asked specific questions about two addends and sometimes she would step back and ask more general questions about the number of ways there are to get to the sum. This flexibility seemed to demonstrate Jennie's growing comfort level with the discursive nature of CGI instruction (Carpenter et al., 2015) and gave evidence for the LPI's influence on Jennie's instructional practices in the second and third lessons.

Summary of Jennie's experiences. The lesson plan intervention successfully equipped Jennie to write lesson plans that translated into many CGI elements being enacted in her instruction. There were marked differences between her first lesson plan and her second and third lesson plans. She integrated significantly more CGI elements into her second and third lesson plans. The investigative elements she integrated revealed the grasp Jenny had on the importance of interpreting students' strategies from a conceptual viewpoint. The kinds of discussions she had with her students after the LPI indicated she was able to practice the elements of CGI instruction that she included in her lesson plans to help her understand what her students were trying to do as they solved problems. It appeared that the LPI helped Jennie elicit and interpret her students' initial strategy offerings from a less procedural framework than her first observed lesson. It appeared that Jennie's openness to conceptually based instruction was a part of her successful integration of the CGI framework. While there were aspects of Jennie's positive personality that may have supported her successful experience of the LPI, the LPI was associated with changes in Jennie's integration of CGI elements into her lesson plans that affected how she approached her students' initial mathematical responses and follow-up reflections during problem-solving lessons. Next is a look at Penny's experiences with the LPI.

Penny and the LPI

Penny's engagement in the LPI was influential in changing her practices in writing CGI framework-based lesson plans. However, the LPI's effects on Penny's instruction was less substantial. Though her second enacted lesson revealed many instances of Penny successfully eliciting and interpreting student mathematics strategies, her third enacted lesson was mostly a return to her pre-LPI practices, with more direct instruction and less follow up of student initial responses.

Research question one. Research Question One asked, “What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI (lesson plan intervention)?” The LPI had a substantive effect on Penny’s lesson planning. Penny integrated many CGI elements into all of her lesson plans, but more elements after the LPI, especially elements that *invited students* to talk about their mathematical thinking. These included: *invites students to reflect, invites multiple strategies, willing to struggle* to understand student thinking, and *pursues thinking* to conclusion. Penny’s lesson plans changed to increase the chances that her first graders would be invited to share their mathematical thinking with each other and the class. Lesson planning can be difficult for beginning teachers, especially aligning objectives with appropriate activities (Darling-Hammond et al., 2005). The LPI was designed partly to remediate this issue, helping Penny write lesson plans that, when enacted, would effectively elicit and interpret student thinking for sense making.

The first LPI session appeared to help Penny understand how specific solution strategies coordinated mathematically with problem types. This occurred through the discussions that occurred during the videos of students solving problems. The first session also included discussions of the CGI solution strategy chart. Penny commented several times that CGI instruction in the methods course was difficult and confusing for her, but became clearer during the LPI sessions. I speculate that the clarification came from the back and forth dialogue that emerged in the first session, where Penny and I explored students’ common solution strategies in the contexts of the mathematics problems they solved. This appeared to be a strength of the LPI, making discussion spaces for PSETs to explore how CGI elements, problem solving, and student thinking were related.

The first session seemed to help Penny understand the levels of mathematical sophistication children have by the solution strategies they chose. It seems reasonable that this occurred because the first LPI session contained many detailed conversations of students' mathematical thinking about problem solving. Unpacking these vignettes with Penny elicited several "Holy Guacamole's" from her, as she saw how students used three kinds of invented algorithms: incremental, combining same units, and compensating to solve word problems. Penny's knowledge of student mathematical capacities was expanded, allowing her to have a better stance at interpreting student thinking to build their number sense. This finding is similar to Philipp et al.'s (2007) findings about PSETs' increased sophistication of mathematical beliefs while observing videos of children solving problems.

Penny's second plan made room for mathematical discussions through questions designed to elicit student mathematical thinking. When writing the second lesson plan, Penny included many questions she would ask students, most of which were designed to elicit student thinking and student reflections on other students' thinking. These questions were co-created and part of the second LPI session. The second session also included discussions about the kind of CGI framework problem types she could include in her lesson plan and the number sets that would be appropriate for her group.

In the second LPI session, Penny mentioned that the pedagogy she was using in her class, what she called, "I do, we do, they do" was antithetical to CGI instruction.

Penny stated that students needed to be given mathematics information and that CGI framework's idea of students' inherent knowledge was new to her. During the LPI sessions, Penny stated how her feelings about CGI practices had changed since the mathematics methods course (where it was introduced):

> I'm just amazed how frustrating it was for me in the junior year compared to now. I keep thinking back to math methods [which the researcher taught] being frustrated, and being like, "Why is he showing this to me? This is confusing math. I'm never going to use this." In my mind I was really like counting this as a loss. But it's crazy what a couple months in a classroom setting can do, what kids think of. My perspective already has changed because I do underestimate the students a lot.

Penny's student teaching experiences opened her mind to appreciate her first graders' intuitive problem solving capacities. These experiences in turn set her in a good place to better understand and utilize the LPI's framework of CGI elements to elicit, interpret, and leverage her students' thinking. In one study, a scaffolding of preservice teachers to elicit and interpret student mathematical thinking was found to be differentially supportive (Sleep & Boerst, 2012). They revealed that practice-based scaffolds were helpful for beginning teachers and that, with some preservice teachers, "additional conceptual and metacognitive scaffolding could have enhanced intern's practice and supported their understanding of the components and rationales for the practice" (Sleep & Boerst, 2012, p. 1046). With this idea in mind, the use of the LPI as a scaffold needed to address a balance of practice-based pedagogies along with the theoretical principles and elements of CGI framework in order for the PSETs to have a reasonable chance at implementing CGI framework ideas from their lesson plans to their instruction.

Research question two. Research Question Two asked, "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" Four elements emerged in both her second and third lessons that were not in Penny's first lesson: *invites multiple strategies, poses problems without modeling, willing to struggle*, and *intentional listening.* The LPI was associated with an increase in the number of CGI elements Penny enacted in her second lesson (eight more) and third lesson (two more) than in her first lesson (seven). These elements made discussion space for students to share their solution strategies and opinions of other students' mathematical ideas. This was a big change from Penny's pattern of directed instruction. However, this increase was mostly associated with her second enacted lesson. In her second lesson, Penny was able to elicit and follow up on student solution strategies more effectively than the other two lessons. It is probable Penny used fewer elements in her third lesson because of her decision to use equations with missing addends instead of posing the word problems that were in her third lesson plan. A discussion of this follows.

In the last interview of the study, Penny asked philosophical questions about the appropriateness of word problems for first graders. She stated that the language of word problems added to the complexity of the mathematics itself and that word problems did not come naturally to them. Penny's statement touches on an important issue in mathematics and language - that students' grasp of the language of mathematics is a complex issue, as is predicting students' mathematical development (Seethaler, Fuchs, Fuchs, & Compton, 2011). Penny seemed to believe that the meaning implied in mathematics symbols would be a simpler task for her students to grasp than the meaning implied by the words that are present in word problems. Penny declared:

> Today I wanted to show them equations that represent problems. That helps their brain with that idea because obviously when we went straight to the word problems there was a lot of disconnect [in her third enacted lesson]. Like kids that even got what was happening today weren't able to get those word problems. My hope was that by seeing it outside of a word problem they might be able to get their brain thinking that way. That when they see it in a word problem it wouldn't be such an abstract idea for them. That they would maybe have some context of what to do in the situation.

Penny was aware of comprehension being an issue with word problems and that the context of a problem was important. However, the word problems were read aloud two to four times by Penny, fairly eliminating any misunderstandings that might be accorded with students reading the problem themselves. Also, Penny spent more time giving context to her third lesson problems through directed discussion and direct instruction than in her second lesson. It seems likely that this occurred because she did not use word problems in her third enacted lesson. Word problems give context to the actions or states of quantities, whereas equations require knowledge of symbols – something that first graders would not likely have experienced. Penny's idea to not use word problems with first graders is a problematic viewpoint mathematically, but reading comprehension is a legitimate issue when children solve word problems (Ulu, 2016). Penny's decision to not enact the word problems she had planned for seemed to have been influenced by her belief that word problems were not cognitively appropriate for her first grade students. As a detail, one day elapsed between Penny writing her third lesson plan and its enactment.

The other CGI element that Penny had planned for but did not enact in her third lesson was *inviting students to reflect*. I speculate Penny did not *invite students to reflect* because she had earlier stated she was sometimes afraid she would not know how to respond accurately to students' spontaneous mathematical reflections. This is a common theme among new teachers, as orchestrating productive mathematical discussions is a difficult practice and takes time to learn (Boerst et al., 2011; Peterson & Leatham, 2009; Stein et al., 2008).

Penny also stated that CGI instruction didn't have enough teacher-centered modeling for her students and this made it unnatural and not concrete enough for them. Penny had the misconception that CGI did not allow for modeling of solution strategies. However, CGI does allow for modeling. CGI instruction works to elicit models of problem solving that start with students' initial ideas. The LPI discussions we had touched on the importance of observing students' physical modeling (usually with cubes) of their solution strategies to determine their level of mathematical sophistication. The LPI discussions seemed to help Penny accurately interpret students' modeling of their solution strategies and to help Penny decide what questions she could ask to compare students' strategies (Carpenter et al., 2015). I speculate Penny had a different definition of modeling than I did because I used the term "modeling" to mean repeating or broadcasting a student's thinking for the class to observe, whereas Penny seemed to believe that modeling was what a teacher does to show the class the answer to a problem.

Penny did not pose word problems in her third lesson because of her conflict about the appropriateness of posing word problems to first graders and her apparent misconception that CGI practices did not address the modeling needs of her students.

Student teachers' unresolved conflicts in their beliefs and images of teaching can affect their professional growth (Wiggins & Clift, 1995). Penny's conflict between CGI framework's emphasis on word problems and her belief that her students weren't ready for word problems yet would make it difficult for her to see some elements of CGI practices as good for her first graders. Penny's disposition toward CGI framework was mixed during some parts of the methods course (as stated above), but appeared to change after she spent some time in her student teaching classroom. Penny stated she liked how CGI practices could help her pursue student thinking, but that elements linked to story/word problems were less beneficial because her first graders were not ready for word problems. Penny was an advocate for leveraging student thinking to build number sense, but was conflicted about how to best build context for mathematics problems, preferring symbolic representations (equations) instead of linguistic representations (word problems).

Individual teacher's beliefs affect their mathematical teaching practices (Lui & Bonner, 2016), so it would be normal for Penny to follow her beliefs and not be substantially affected by the CGI framework that the LPI forwarded. Beliefs of preservice teachers have been shown to change (Hart, 2002; Swars, Smith, Smith, & Hart, 2006), but have also been shown to be stable over time, depending on their own internal locus of authority (Cady, Meier, & Lubinski, 2006), and availability of support for more reformed types of practices (Vacc & Bright, 1999). As Penny's case exemplifies, it is also common for PSETs to acknowledge the tenets of CGI framework but not be able to use them in their instruction for a number of reasons (Vacc & Bright, 1999), including needing more time.

In her third enacted lesson, Penny did *invite multiple strategies* from her students and demonstrated a *willingness to struggle* to understand student thinking. This was consistent with her second lesson and demonstrated a change from her first lesson, which focused on direct instruction. The LPI had some carryover effect for Penny in these two elements. It is possible that this occurred because these elements were a strong point in her second enacted lesson. Perhaps Penny felt these elements were *doable* because they did not require an immediate response from her to the class, something she felt anxious about. Her felt beliefs would be a major factor in attempting to enact favorable practices (Temiz & Topcu, 2013). It is common for preservice teachers to focus on their instruction while teaching and less on responding to students' thinking (Levin et al., 2009).

Research question three. Research Question Three asked, "What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?" In her first lesson, Penny practiced direct instruction, reviewing mathematics vocabulary from previous lessons. Direct instruction is often the default practice for new teachers, especially PSETs (Chazan & Ball, 1999). Her second lesson showed some direct instruction, but mostly she used directed discussion and inquiry-guided discussions. This was a major change in her teaching practice from the first lesson. It was impressive for Penny to have tried a new teaching strategy after just two CGI framework training sessions. It is common practice for preservice teachers to teach in ways they are familiar with, and they often struggle to change their instructional practices when given opportunities to enact new pedagogical strategies (Ball, 1988).

Her third lesson drew mathematical strategies from her students using direct instruction and directed discussion. She asked a few students follow-up questions and then usually gave her interpretation of the child's (unspoken) strategy. Much of her speaking was lecture oriented, but with CGI elements mixed in, such as *broadcasting* a student's initial response. Penny's third lesson did have more student-centered dialog, but was sometimes truncated. Like Jennie, Penny seemed to be trying her best to integrate familiar pedagogies with the less familiar pedagogy of the CGI framework (Ball, 1988).

Differences emerged between Jennie and Penny's third enacted lessons. Penny was more likely to use direct instruction as a strategy than Jennie. Penny asked students to "explain" their strategies more than Jennie. Penny also used more visual models, even if they were not accurately analyzed in her explanations. For instance, Penny stated that "our answer on a math mountain is always on the top of the mountain", when the answer to the question was actually one of the bottom numbers (addends) on the mountain.

Jennie and Penny were similar in their third lessons as both returned to using symbolic representations (equations and expressions) in some of their instruction, as they did in their first lessons. It seems they were more comfortable with demonstrating mathematical strategies using symbols than with more linguistic forms. However, they both asked students more questions for each problem posed than in their first lessons, showing differences in their third lesson pedagogical approaches from their first lessons, these differences being associated with the LPI.

Summary of Penny's experiences. The LPI had an effect on Penny's lesson planning in student problem solving, but did not have a lasting effect on her instructional practices after the second lesson. Penny's second enacted lesson elicited more student responses than her first lesson, and Penny followed up on more student solution strategies than in the other two lessons. The LPI sessions revealed that Penny significantly increased her practical understanding of CGI framework from the previous mathematics methods course, as evidenced in the LPI sessions conversations. It seems probable that Penny's opinion about word problems being inappropriate for first graders affected her instruction. It also seems probable that another opinion affected Penny's instructional practices, that symbolic forms of mathematics problems (equations) were better than word problems at communicating the operations of addition and subtraction. Penny's preference for visual models (math mountains) and her sometimes incorrect interpretation of these models complicated her conversations with her students. While it cannot be said Penny wholly returned to her first lesson practices, Penny's preference for direct instruction in her third enacted lesson was inconsistent with the investigative nature of CGI instruction and was a place of conflict in Penny's instructional practices. Next is a look at Eleanor's experience in the LPI.

Eleanor and the LPI

The LPI had an influence on Eleanor's lesson plans, but less influence on her instructional practices. Her second and third lesson plans contained additional CGI elements from her first lesson plan, but not significantly more. The goals she had for her second and third lesson plans were more CGI framework oriented, but only slightly more. The LPI was associated with a small increase in integrated CGI elements in her second and third enacted lessons. Changes in her instruction were mostly her *pursuit to completion* of a problem with individual children. Eleanor's pedagogical practices after the LPI were similar to those before the LPI.

Research question one. Research Question One asked, "What elements of CGI framework do PSETs integrate into early number lesson plans constructed before, during, and after an LPI?" Eleanor's first lesson plan incorporated 8 elements, the most common being *invites students to share*. Her first lesson plan was the most complex and ambitious in the study, even asking first graders to write their own story problems. Eleanor's second and third lesson plans both added two identical elements, *invites students to reflect* on other students' thinking and *presents problems without modeling*. The change seemed to be minor in that these two elements seemed to be an extension of Eleanor's general expectations for students - that children are capable thinkers and should be expected to contribute during large group instruction. This was demonstrated in her first lesson as she had many open ended and analytical questions like, "What do you notice that is the same about each problem?"

In the LPI sessions, Eleanor was quick to see how CGI elements could help her illuminate student solution strategies and readily merged CGI elements into her second lesson plan. She also made statements about levels of difficulty inherent in the problem types and was knowledgeable of common solution strategies that fit each type (using the CGI framework solution strategies chart, Appendix F). Before the study began, Eleanor had been studying CGI elements on internet videos and seemed interested in letting students choose their own solution strategies. Eleanor was adept at interpreting students' solution strategies in the training videos (of students solving problems). I speculate that Eleanor's ability to interpret students' strategies on the videos might have been due to the first session of the LPI, where we reviewed the chart of problem types and their corresponding solution strategies.

As Eleanor and I co-wrote the lesson plan, Eleanor referred back to some of the conversations we had during the first LPI session. These conversations were about how to match elements that fit into specific word problems and solution strategies, demonstrating Eleanor's curiosity and a solid understanding of the CGI framework. In the LPI sessions, Eleanor appeared to see CGI practices not as a protocol of things to do when problem solving, but as elements to investigate children's thinking. I speculate that the LPI had some effect on Eleanor's lesson planning because the LPI laid out specific vocabulary of problem types, solution strategies, and sophistication levels of student thinking. These CGI framework terms seemed to be adopted by Eleanor in our LPI discussions and in subsequent post-lesson conversations and interviews.

Eleanor's third lesson plan was similar to her second in CGI framework elements present. Her third lesson plan revealed *flexible range of numbers*, whereas her second lesson plan revealed *intentional listening to student thinking.* Eleanor's third lesson plan was different from her second in that she added a different (and more difficult) problem type as well as adding extra number sets that were to be used with this more difficult problem type. The LPI's integration of CGI content was associated with Eleanor's self-initiated integration of more difficult mathematics word problems and multiple number sets. I speculate Eleanor added the more difficult problem type because she had stated earlier that she wanted to try different problem types with her students. Eleanor's inclusion of extra number sets for her word problems demonstrated some of her mathematics pedagogical content knowledge (Ball, Thames, & Phelps, 2008). Her use of multiple number sets with identical problems set up a learning space for her students to explore how strategies and quantities are related. Capraro, Capraro, Parker, Kulm, and Raulerson (2005) reported that "mathematically competent preservice teachers exhibited progressively more pedagogical content knowledge as they were exposed to mathematics pedagogy during their mathematics methods course" (p. 101). This connects to Eleanor's growth in understanding how CGI practices can be used to question students specifically when a student first offers a solution strategy.

Research question two. Research Question Two asked, "What elements of CGI framework do PSETs enact while they teach early number lessons constructed before, during, and after an LPI?" The LPI appeared to influence Eleanor to enact more CGI elements into her second lesson (15) and third lesson (11) than in her first lesson (7). Though her first lesson revealed Eleanor repeatedly asking, "Is there a different way we could do this?" she did not use other CGI framework-based elements to consistently follow up on student thinking. Eleanor (like Jennie and Penny) enacted more elements in her second lesson than what she included in her lesson plans. In her second lesson, Eleanor asked specific and general follow-up questions that were conceptual in nature. With CGI framework's emphasis on conceptual understanding (not just procedural) Eleanor was potentially positioned to take more advantage of the LPI than a less mathematically concept-oriented PSET (Soto-Johnson et al., 2008). This was demonstrated during the LPI sessions as Eleanor asked many questions that were conceptual in nature, such as: why certain problem types were harder than others, how CGI framework solution strategies might be related to reading strategies, and whether the categories of solution strategies pointed to certain age students. Eleanor's analysis of how CGI practices were framed seemed to indicate that she approached CGI framework pedagogically - not as a set of procedural questions, but as a way of looking at how a model of teaching mathematics could help her understand what her students were thinking as they solved word problems. Several times during LPI sessions Eleanor asked what the purpose of some elements were or whether the goal of CGI framework was to use manipulatives, etc. All of this data seemed to point to Eleanor's effective use of CGI elements as a mediator for student mathematical development. Relevant literature points to new teachers having varied levels of success utilizing CGI elements and that there are many factors affecting these successes (Carpenter et al., 2015).

Some CGI elements emerge in PSETs' instruction after they realize students can stay focused on the mathematics being discussed. After the second lesson, Eleanor stated that she was surprised to see how long students would track with a specific problem conversation. A snippet from the conversation revealed:

> I thought, wow, they still want to stay on this problem! We've been on this problem for the last 20 minutes and, I don't know how long it was, 10 minutes, 15 minutes, I'm really surprised that they are disappointed having to erase their board. I would think by now it would be like, "Oh, thank goodness, she's moving on to the next problem."

Eleanor's third lesson practices included intentional invitations to share, but only if students had new solution strategies for a posed problem. Eleanor was the only PSET to ask for different solutions to the same problem. Eleanor also gave large group time to do partner sharing - students sharing their solution strategies with a partner. As an instructor, Eleanor had positioned her students as experts (Gadanidis, Hughes, & Borba, 2008), an important aspect of CGI instruction and evidence that the LPI may have helped Eleanor enact this pedagogy. I speculate that Eleanor's invitation for new or unique solution strategies was part of her demeanor or pedagogical preference, as she stated in an interview that she liked to try different ways to solve problems, looking for patterns that might emerge from the problem.

Research question three. Research Question Three asked, “What teaching practices do PSETs demonstrate before, during, and after an enacted LPI lesson?” Eleanor practiced directed discussion in her first lesson, mostly guided inquiry in her second lesson, and a mixture of directed discussion and guided inquiry in her third lesson. Eleanor’s instruction varied the least of all three PSETs, but was the most child-centered, eliciting (comparatively) more student mathematical thinking across all three lessons. I speculate that this could be due to her confidence in her first graders’ mathematical capacities, demonstrated in one of her interviews. Eleanor stated, “He knew the answer in his head but he never wrote it down. It's interesting when you look at these [worksheets]; they have so many different ways of doing things.” It also seems reasonable that Eleanor’s teaching practices would not substantively change if she believed her instruction prior to the LPI was similar to CGI framework.

Summary of Eleanor’s experiences. The LPI was associated with mixed responses on Eleanor’s lesson planning and instruction. Eleanor’s second and third lesson plans were similar from a CGI framework perspective. The LPI session discussions revealed that Eleanor had high expectations for her students, coinciding with her stated beliefs that her students could solve word problems, even though they were first graders. Her third lesson plan was different from her second in that she included extra number sets for word problems and that she intentionally sought out multiple strategies for each problem. The LPI was associated with an increase in integrated CGI elements in her second and third enacted lessons. Eleanor’s instructional practices indicated that she approached CGI framework *as* a framework and not as a protocol or a list of procedures. Despite Eleanor’s belief that her students could solve her problems, she was still surprised that they could engage in extended discussions on single problems. Eleanor asked the most questions of her students of the three PSETs, but her instructional practices were not significantly different after the LPI.

Conclusions

The effects of the LPI in three PSETs were mixed, although after the LPI, all three experienced some successes in utilizing CGI framework elements in their lesson planning and instruction. Variations in their lesson planning and teaching practices emerged, as was anticipated. The quantity and specificity of follow up questions also varied by PSET, resulting in differences in revealed student thinking and depth of solution strategy exploration.

In five of the six post-LPI enacted lessons, the PSETs utilized more CGI elements than in the corresponding lesson plan, with the exception of Penny's third enacted lesson. I speculate that Jennie and Eleanor enacted more CGI elements than what they had planned for because of their increasing comfort with asking students follow up questions, which encapsulates a subgroup of CGI elements. It is not uncommon for elementary lessons to involve more teaching techniques, discussions, and activities than what is mentioned in a lesson plan.

All three PSETs experienced instances of student's intuitive sense of strategies, where strategies did not come from modeling procedures to students. The PSETs experienced their students making sense of problems, even though discussion sessions took time to elicit. The PSETs practiced the element of *intentional listening* and thus were able to interpret students' initial and secondary responses (Barnhardt & van Es, 2015; Levin et al., 2009; Peterson & Leatham, 2009; Stockero, Peterson, Leatham, & Van Zoest, 2014).

The LPI was most successful for Jennie, whose openness to new ways of teaching and conceptually based disposition seemed to situate her well for the LPI. Jennie's pliability was demonstrated in her lesson planning and enacted instruction. Her enthusiasm and intentionality for integrating CGI principles in the LPI seemed to have leveraged her curiosity from the CGI knowledge she gleaned from the researcher's previous mathematics methods course. Jennie's curiosity seemed to overcome her sense that she was not a good "math person". Jennie demonstrated grit (Duckworth, 2016) in her demeanor and her efforts in the study.

The LPI was least successful for Penny, whose pedagogy was most often procedural instruction. Penny's second and third lesson plans were oriented well for CGI instruction. Her second lesson exhibited a pattern of posing a problem, inviting students to share, listening to a strategy, broadcasting the language of the child, and then asking two or three follow up questions without waiting for answers. Several times Penny did ask specific follow up questions pertinent to the solution strategy offered, but struggled with follow up questions about exactly what the child did. Penny consistently jumped in to finish explaining solution strategies to the class. She did not follow up on students' reflections on solution strategies, nor did she ask students to compare strategies to build number sense. Penny's third enacted lesson was mostly a return to her first lesson, albeit with more CGI elements. She used equations instead of word problems, as she felt word problems were not appropriate for her class. Penny's lower expectations for students' innate ability to comprehend and solve word problems seemed to inhibit her rigor for discussions that elicited student thinking. As CGI framework is not a curriculum or set of procedures for teaching, using CGI framework in an LPI would require scaffolding to leverage its tenets with a student teacher with tendencies toward procedural instructional practices.

Eleanor's experience with the LPI was mixed. Her lesson planning and instruction utilized more CGI elements after the LPI. Eleanor asked many specific and general follow up questions after the LPI, resulting in elicited student thinking. Her second enacted lesson exhibited many rich conversations about how students solved their problems as Eleanor asked specific follow up questions, demonstrating the CGI element of *pursues to completion.* Eleanor's third lesson was a partial return to her first enacted lesson as she practiced more direct instruction than in her second lesson.

Eleanor's overall stance as a teacher was the most like a facilitator, and therefore amenable to CGI framework. Her first lesson demonstrated Eleanor's practice of keeping students at the center of discussions and situating them more often as experts, both of which make it harder to determine if the LPI had an effect on her lesson planning and instruction. Eleanor demonstrated conceptual thinking, making CGI practices a good fit for her pedagogically. Of the three participants, Eleanor was perhaps the least in need of a bridge between a mathematics methods course and her mathematics instruction during her student teaching. Eleanor's natural disposition for teaching was to ask lots of questions, and this pedagogy fits well with CGI framework and practices. She was more than tolerant of some awkward moments during instruction, welcoming students' natural spontaneous responses during discussions. The LPI's use of CGI framework seemed to help Eleanor organize her word problem lessons and gave her a more detailed idea of how solution strategies indicate mathematical sophistication in student thinking.

Importance of Study

Bridging methods course content and pedagogies into PSETs' classrooms is an important and difficult step (Valencia et al., 2009) in their PSET teaching journey (Bell & Robinson, 2004). As a form of PSET field support material (Hertzog & O'Rode, 2011) and scaffold (Rusznyak & Walton, 2011) this lesson plan intervention was an effective tool in bridging methods coursework in CGI framework with field placement instruction. Although lesson plans can be difficult for PSETs to create (Drost & Levine, 2015), they are effective tools for teaching and are uniquely suited for translating important content material into PSETs' classrooms (Rusznyak & Walton, 2011).

As the PSETs in this study engaged in their first extended teaching responsibilities (12 weeks), they benefited from the LPI as it scaffolded CGI material into lesson plans that proved effective at integrating mathematics instruction about problem solving (Rusznyak & Walton, 2011). This LPI revealed that PSETs have different capacities for utilizing a lesson plan intervention to integrate mathematics methods coursework into their student teaching classrooms. The different capacities that emerged from the LPI are important for teacher educators engaged in supporting PSETs as they teach mathematical problem solving in their student teaching classrooms. One of the qualities the LPI had was to reconnect PSETs with previously learned content and pedagogies that may not have transferred to their student teaching placement classrooms without scaffolding (Darling-Hammond, 2014). The conversational nature of the LPI helped PSETs address important issues that arose in their particular classroom settings.

As we met for the training sessions, PSETs' lesson goals were individually blended with CGI elements to best meet their students' unique needs when engaged in problem solving exercises. Since each lesson plan had unique objectives, the LPIs proved to be flexible enough to accommodate these objectives while integrating CGI elements to elicit student thinking. The conversational nature of the LPI also helped the PSETs build lesson plans that supported the unique dynamics that every elementary classroom has. These dynamics included: different capacities of their first graders to handle whole class discussions, cooperating teachers' disposition and skill with CGI practices, PSETs' content and pedagogical skills, instructional materials, and other learning environment issues that are relevant to classroom whole-group instruction.

Several components of the LPI seemed valuable for equipping the three PSETs to elicit and utilize student mathematical thinking. The first session of the LPI gave all of the PSETs a review of how students think mathematically as they solve problems. It also gave them specific connections between problem types and commonly used solution strategies (Appendices E and F). The first LPI session was crucial for reviewing and re-teaching the overarching principles of CGI framework as well as its specific framework for understanding student thinking - the key ingredient to helping PSETs leverage children's thinking to build their number sense.

The second session of the LPI afforded meaningful mathematical conversations between the PSETs and the researcher. These included the mathematical challenges that children might have and how these gave evidence of students' mathematical sophistication and conceptual understanding. In the second session, content emerged about how to integrate CGI elements into their lesson plans. This content included discussions about what follow up questions they could ask students based on the problem type and children's initial responses. The second session was helpful to the PSETs through the many topics that occurred semi-spontaneously, including: PSET expectations of student responses, appropriate number set choices, specific CGI elements designed to elicit student thinking, how to interpret student responses, and how to orchestrate conversations so that mathematical concepts and strategies could emerge.

The LPI framework also afforded some unintended benefits to PSETs' efforts to elicit, interpret, and leverage student thinking in their third lesson plan and third enacted lesson. This came about through the post-lesson conversations (right after the enacted lessons) and interviews (conducted at the end of the day). It was not anticipated that the PSETs' reflections afforded by these semi-structured discussions would open conversations about how to improve the third lesson plan, but as I followed up their initial responses our conversations seemed to stimulate thoughts about how their third lesson plan and lesson could be improved.

Suggestions for Future Practice

Some components not included in the LPI would have improved PSETs' capacities to elicit, interpret, and leverage student thinking. A component that would benefit the LPI would be training about how to notice students' strategies while teaching in large groups (Jacobs et al., 2010). This would have enabled the PSETs to be better prepared to notice and interpret students' responses in real time. Adding some notes about this in their lesson plans would have equipped them to better anticipate and respond to students' responses (Peterson & Leatham, 2009). A third recommended component involves a review of videos of the PSETs' first enacted lesson. This would give immediate feedback to PSETs (Schwartz et al., 2018) about follow up questions, mathematical concept noticing and interpreting, and missed opportunities to pursue student thinking. It would be beneficial to add training about recognizing and addressing common student misconceptions while teaching in real time. This is a difficult skill and would have helped the PSETs pursue students' mathematical thinking to make connections between strategies. Finally, finding ways to scale up this LPI model to meet the needs of a whole class of PSETs would be worthwhile.

Directions for Further Research

Based on the results of this study, it would be beneficial to explore another CGI framework-based LPI with a larger group of PSETs to determine if the suggested changes in training would increase their capacities to elicit and utilize student mathematical thinking. Another study could explore how CGI elements in elementary mathematics methods courses could be taught so that CGI practices could be better integrated into PSETs' student teaching placements. Since many of the PSETs' mathematical discussions about individual problems did not summit in conceptual thinking being explored with students, a study could pursue how PSETs could be equipped with an LPI to better follow up mathematical conversations with specific questions to accomplish more instances of *pursue students' thinking to completion*. On reflection, since all three of the PSETs demonstrated capacities to incorporate CGI elements into their lesson plans with a mentor's help, a study could explore the nuances of cooperating teachers' efforts to help PSETs build lesson plans that elicit and support student thinking for problem solving. Also, a triadic approach to helping PSETs integrate CGI framework-based lesson plans would be helpful in a future study. This approach would better coordinate

PSETs with their university supervisors and cooperating teachers to plan and enact CGI elements for problem solving (Valencia et al., 2009).

Since CGI framework is an effective way to understand students' mathematical thinking, a study could explore the elements of CGI framework with regard to teacher noticing of student mathematical discourses (Barnhardt & van Es, 2015). Stein et al.'s (2008) work on orchestrating productive mathematical discussions could be integrated in a study with select elements of CGI instruction to see if PSETs could benefit from the conversational aspects of both CGI framework and Stein et al.'s (2008) practices of orchestrating mathematical discussions.

Japanese lesson plan study (Hart, Alston, & Murata, 2011) would be a natural setting to understand how integrating CGI framework into lesson plans could benefit PSETs attempting to teach word problems to young learners. Lesson study can be a complex task for PSETs, but could bring multiple viewpoints to their lesson planning that would not otherwise be accomplished.

The elements and pedagogical aspects of the CGI framework have been a great resource to preservice and in-service teachers alike. I look forward to other researchers' efforts in the areas of CGI framework, PSET development, and student teaching.

References

Abell, S. K., & Bryan, L. A. (1997). Reconceptualizing the elementary science methods course using a reflection orientation. Journal of Science Teacher Education, 8, 153–166.

Amador, J., & Weiland, I. (2015). What preservice teachers and knowledgeable others professionally notice during lesson study. Teacher Educator, 50(2), 109–126. doi.org/10.1080/08878730.2015.1009221

Ambrosetti, A., & Dekkers, J. (2010). The interconnectedness of the roles of mentors and mentees in pre-service teacher education mentoring relationships. *Australian Journal of Teacher Education*, *35*(6). doi.org/10.14221/ajte.2010v35n6.3

Bahr, D. L., & Monroe, E. E. (2008). An exploration of the effects of a practicum intensive mathematics methods course on the beliefs of elementary preservice teachers. *International Journal of Mathematics Teaching and Learning*, *9*(2), 297. http://www.cimt.plymouth.ac.uk/journal/ default.htm.

Baker, K., & Harter, M. E. (2015). A living metaphor of differentiation: A meta-ethnography of Cognitively Guided Instruction in the elementary classroom. *Journal of Mathematics Education at Teachers College*, *6*(2), 27-36.

Baldinger, E. E., & Lai, Y. (2019). Pedagogical context and proof validation: The role of positioning as a teacher or student. *Journal of Mathematical Behavior, (in press).* doi-org.ezp1.lib.umn.edu/10.1016/j.jmathb.2019.03.005

Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*. *8*(1), 40-48.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, *93*(4), 373-397.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, *60*(5), 497-511.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29(*1), 14-17, 20-22, 43-46. Retrieved from http://hdl.handle.net/2027.42/65072

Ball, D. L., & Peoples, B. (2007). Assessing mathematical proficiency. In Alan Schoenfeld (Ed.), *Assessing a student's mathematical knowledge by way of interview* (pp. 213-268). NY: Cambridge University Press.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education. 59*(5), 389-407.

Barnett, B. G. (1995). Developing reflection and expertise: Can mentors make the difference? *Journal of Educational Administration*, *33*(5), 45-59. doi.org/10.1108/09578239510098527

Barnhart, T., & van Es, E. (2015). Studying teacher noticing: Examining the relationship among pre-service science teachers' ability to attend, analyze, and respond to student thinking. *Teaching and Teacher Education*, *45*, 83–93. doi.org/10.1016/j.tate.2014.09.005

Battista, M. T., Baek, J. M., Cramer, K., & Blanton, M. L. (2017). *Reasoning and Sense Making in the Mathematics Classroom, Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics.

Bell, C. L., & Robinson, N. G. (2004). The successful student-teaching experience: Thoughts from the ivory tower. *Music Educators Journal*, *91*(1), 39-42.

Boaler, J. (2015). *What's math got to do with it? How teachers and parents can transform mathematics learning and inspire success.* New York: Penguin.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* San Francisco, CA: Josey-Bass.

Boerst, T. A., Sleep, L., Ball, D., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record*, *13(*12), 2844–2877. Retrieved from http://www.tcrecord.org

Bright, G. W., Behr, M. J., Post, T. R., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education*, *19*(3), 215-232.

Brophy, J. E., & Good, T. L. (1970). Teachers' communication of differential expectations for children's classroom performance: Some behavioral data. *Journal of Educational Psychology*, *61*(5), 365-374.

Brown, C. A., & Borko, H. (1992). Becoming a mathematics teacher. In D. Grouws (Ed.) *Handbook of research on mathematics teaching and learning*, (pp. 209-239). NY: Simon & Schuster.

Bullock, D. (2004). Moving from theory to practice: An examination of the factors that preservice teachers encounter as they attempt to gain experience teaching with technology during field placement experiences. *Journal of Technology and Teacher Education*, *12*(2), 211–237.

Cady, J., Meier, S., & Lubinski, C. (2006). Developing mathematics teachers: The transition from preservice to experienced teacher. *Journal of Educational Resources, 99*(5), 295-305.

Capraro, R. M., Capraro, M. M., Parker, D., Kulm, G., & Raulerson, T. (2005). The mathematics content knowledge role in developing preservice teachers' pedagogical content knowledge. *Journal of Research in Childhood Education*, *20*(2), 102-118. doi:10.1080/02568540509594555

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education, 24*(5), 428–441.

Carpenter, T. P., & Fennema, E. (1992a). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research, 17,* 457-470.

Carpenter, T. P., & Fennema, E. (1992b). Cognitively guided instruction: Building on the knowledge of students and teachers. In W. Secada (Ed.), *Researching educational reform: The case of school mathematics in the United States* (pp. 457-470). Special Issue of International Journal of Educational Research. New York: Elsevier.

Carpenter, T. P., Fennema, E., Franke, M. L., Empson, S., & Levi, L. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., Empson, S. (2015). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., & Loef-Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, *97*(1), 3–20.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499–531.

Carpenter, T. P., & Franke, M. L. (2004). Cognitively guided instruction: Challenging the core of educational practice. In Thomas Glennan, Susan Bodily, Jolene Galegher and Kerri Kerr (Eds.) *Expanding the reach of education reforms: Perspectives from leaders in the scale-up of educational interventions* (pp. 41-80). Santa Monica, CA: Rand.

Carpenter, T. P., Franke, M. L., Jacobs, V. R., Fennema, E., & Empson, S. B. (1998). A longitudinal study of invention and understanding in children's multidigit addition and subtraction. *Journal for Research in Mathematics Education*, *29*(1), 3–20.

Carpenter, T. P., Franke, M. L., Johnson, N. C., Turrou, A. C., & Wager, A. A. (2017). *Young children's mathematics: Cognitively guided instruction in early childhood education*. Portsmouth, NH: Heinemann.

Carpenter, T. P., & Levi, L. (2000). Developing conceptions of algebraic reasoning in the primary grades. Res. Rep. 00-2). Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science.

Carrier, S. J., (2011). Implementing and integrating effective teaching strategies including features of lesson study in an elementary science methods course. *Teacher Educator*, *46*(2), 145-160.

Carrier, S. J. (2012). Elementary preservice teachers' science vocabulary: Knowledge and application. *Journal of Science Teacher Education, 24*(2), 405-425.

Carter, K., & Richardson-Koehler, V. (1989). Toward a curriculum for initial year of teaching programs. *Elementary School Journal*, *89*(4), 405-420.

Chapin, S., O'Connor, C., & Anderson, N. (2013). *Talk moves: A teacher's guide for using classroom discussions in math.* Sausalito, CA: Math Solutions.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics*, *19*(2), 2–10.

Clark, C. N., and Yinger, R. J. (1987). Teacher planning, in J. Calderhead (Ed.), *Exploring teachers' thinking*. London: Cassell.

Cochran-Smith, M., & Villegas, A. (2015). Framing teacher preparation research: An overview of the field, part 1. *Journal of Teacher Education, 66*(1), 7-20.

Coffey, D. C., (2004). Using alternative assessment to affect preservice teachers' beliefs about mathematics. In T. Watanabe & D. R. Thompson (Eds.), *The work of mathematics teacher educators: Exchanging ideas for effective practice* (AMTE Monograph Series 1, (pp. 49-66). San Diego, CA: Association of Mathematics Teacher Educators.

Cohen, L., Manion, L., & Morrison, K. (2011). *Surveys, longitudinal, cross-sectional, and trend studies: Research methods in education (*7th edition). Abingdon: Routledge.

Cramer, K. A. (2003). Using a translation model for curriculum development and classroom instruction. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 449–463). Mahwah, NJ: Lawrence Erlbaum Associates.

Cramer, K. A., Ahrendt, S., Monson, D., Wyberg, T., & Colum, K. (2017). Fractions, number lines, and third graders. *Teaching Children Mathematics, 24*(3), 190-199.

Cramer, K. A., Post, T. R., & DelMas, R. C. (2002). Initial fraction learning by fourth and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education*, *33*(2), 111-144. doi.org/10.2307/749646

Crespo, S. (2000). Seeing more than right and wrong answers: Prospective teachers' interpretations of students' mathematical work. *Journal of Mathematics Teacher Education 3*(2), 155–181.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches.* Los Angeles, CA: Sage

Darling-Hammond, L. (2010). Teacher education and the American future. *Journal of Teacher Education, 61*(1-2), 35-47.

Darling-Hammond, L. (2014). Strengthening clinical preparation: The holy grail of teacher education. *Peabody Journal of Education, 89*(4), 547–561. doi.org/10.1080/0161956X.2014.939009

Darling-Hammond, L., Banks, J., Zumwalt, K., Gomez, L., Gamoran Sherin, M., Griesdorn, J., & Finn, L. (2005). Educational goals and purposes: Developing a curricular vision for teaching. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 169–200). San Francisco, CA: Jossey-Bass.

Darling-Hammond, L., Hammerness, K., Grossman, P., Rust, F., & Shulman, L. (2005). The design of teacher education programs. In L. Darling-Hammond & J. Bransford (Eds.), *Preparing teachers for a changing world: What teachers should learn and be able to do* (pp. 390-441). San Francisco, CA: Jossey-Bass

Darling-Hammond, L., Richardson, N. (2009). What matters. *Educational Leadership, 66*(5), 46-53.

Davenport, J., & Smetana, L. (2004) Helping new teachers achieve excellence. *The Delta Kappa Gamma Bulletin, 70*(2).

Denzin, N. K., & Lincoln, Y. S. (2000). *The SAGE handbook of qualitative research* (2nd ed.). Thousand Oaks, CA: Sage.

Donche, V., & Van Petegem, P. (2009). The development of learning patterns of student teachers: A cross-sectional and longitudinal study. *Higher Education, 57*(4), 463-475.

Drost, B., & Levine, A. (2015). An analysis of strategies for teaching standards-based lesson plan alignment to preservice teachers. *Journal of Education, 195*(2), 37-47.

Duckworth, A. (2016). *Grit: The power of passion and perseverance.* New York, NY: Scribner.

Dymond, S. K., Bentz, J. L. (2006). Using digital videos to enhance teacher preparation. *Teacher Education and Special Education: The Journal of the Teacher Education, Division of the Council for Exceptional Children, 29*(2), 98-112. doi.org/10.1177/088840640602900202

Edwards, A., & Protheroe, L. (2003). Learning to see in classrooms: What are student teachers learning about teaching and learning while learning to teach in schools? *British Educational Research Journal*, *29*(2), 227–242. doi.org/10.1080/0141192032000060957

Fehn, B., & Koeppen, K. E. (1998). Intensive document-based instruction in a social studies methods course and student teachers' attitudes and practice in subsequent field experiences. *Theory and Research in Social Education*, *26*(4), 461–484. doi.org/10.1080/00933104.1998.10505861

Fennema, E., Carpenter, T. P., Franke, M. L., & Carey, D. A. (1992). Learning to use children's mathematics thinking: A case study. In C. Maher & R. Davis (Eds.), *Relating schools to reality* (pp. 93-118). Needham Heights, MA: Allyn Bacon.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403–434.

Fennema, E., Franke, M., Carpenter, T., & Carey, D. (1993). Using children's knowledge in instruction. *American Educational Research Journal, 30*(3), 555-583.

Ferraro, J. M. (2000). Reflective practice and professional development. Washington, DC: *ERIC Clearinghouse on Teaching and Teacher Education*. Retrieved from http://www.ericfacility.net/ericdigests/ed449120.html

Frade, C., Acioly-Regnier, N., & Jun, L. (2013). Beyond deficit models of learning mathematics: Socio-cultural directions for change and research. In M. Clements, A. Bishop, C. Keitel-Kreidt, J. Kilpatrick, F. K. Leung (Eds.), *Third international handbook of mathematics education* (pp. 101-144). New York, NY: Springer-Verlag.

Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory into Practice, 40*(2), 102-109.

Franke, M. L., Webb, N., Chan, A., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Gadanidis, G., Hughes, J., & Borba, M. (2008). Students as performance mathematicians. *Mathematics Teaching in the Middle School, 14*(3), 168-175. Retrieved from http://www.jstor.org.ezp1.lib.umn.edu/stable/41182659

Gess-Newsome, J. (1999). Pedagogical content knowledge: An introduction and orientation. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 3-17). Dordrecht: Kluwer Academic Publishers.

Gill, M. G., Ashton, P. T., & Algina, J. (2004). Changing preservice teachers' epistemological beliefs about teaching and learning in mathematics: An intervention study. *Contemporary Educational Psychology*, *29*(2), 164–185. doi.org/10.1016/j.cedpsych.2004.01.003

Glaser, B. G., & Strauss, A. L. (2017). *Discovery of grounded theory: Strategies for qualitative research.* Philadelphia, PA: Routledge.

Guerrero, J. (2014). Cognitively guided instruction, a professional development approach to teaching elementary mathematics: A case study of adoption and implementation in three rural elementary schools. (Doctoral dissertation). Retrieved from Proquest Dissertation and Thesis (Accession No. 1547944263).

Haefner, L., & Zembal-Saul, C. (2004). Learning by doing? Prospective elementary teachers' developing understandings of scientific inquiry and science teaching and learning. *International Journal of Science Education, 26*(13), 1653-1674.

Hart, L. C. (2002). Preservice teachers' beliefs and practice after participating in an integrated content/methods course. *School Science and Mathematics, 102*(1), 4-14.

Hart, L. C., Alston, A. S., & Murata, A. (2011). *Lesson study research and practice in mathematics education: Learning together.* Dordrecht: Springer.

Heibert, J., & Stigler, J. W. (2004). A world of difference: Classrooms abroad provide lessons in teaching math and science. *Journal of Staff Development, 24*(4), 10-15.

Hendricks, C. (2013). *The effects of Cognitively Guided Instruction on mathematics achievement of second grade children* (Doctoral dissertation). Walden University. Retrieved from Proquest LLC.

Heritage, M., Kim, J., Vendlinski, T. P., & Herman, J. L. (2008). *From evidence to action: A seamless process in formative assessment?* (CRESST Report 741). Retrieved from: https://files.eric.ed.gov/fulltext/ED502625.pdf

Herppich, S., & Wittwer, J. (2018). Preservice teachers' beliefs about students' mathematical knowledge structure as a foundation for formative assessments. *Teaching and Teacher Education, 76,* 242-254.

Hertzog, H. S., & O'Rode, N. (2011). Improving the quality of elementary mathematics student teaching: Using field support materials to develop reflective practice in student teachers. *Teacher Education Quarterly, 38*(3), 89–111.

Hughes, E. K. (2007). *Lesson planning as a vehicle for developing pre-service secondary teachers' capacity to focus on students' thinking.* (Doctoral dissertation). Retrieved from: http://d-scholarship.pitt.edu/10196/1/EKH-ETDHack2.pdf

Hughes, E. K., & Smith, M. S. (2004). Thinking through a lesson: Lesson planning as evidence of and a vehicle for teacher learning. In Poster presented as part of the symposium, "Developing a Knowledge Base for Teaching: Learning Content and Pedagogy in a Course on Patterns and Functions" at the annual meeting of the American Educational Research Association, San Diego, CA.

Huinker, D., & Madison, S. K. (1997). Preparing efficacious elementary teachers in science and mathematics: The influence of methods courses. *Journal of Science Teacher Education, 8*(2), 107-126.

Jacobs, V., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

John, P. D. (2006). Lesson planning and the student teacher: Re-thinking the dominant model. *Journal of Curriculum Studies, 38*(4), 483-498. doi:10.1080/00220270500363620

Kaminski, E. (2003). Promoting pre-service teacher education students' reflective practice in mathematics. *Asia-Pacific Journal of Teacher Education, 31*(1), 21–32. doi.org/10.1080/13598660301619

Kong, S. C. (2010). Using a web-enabled video system to support student-teachers' self-reflection in teaching practice. *Computers and Education*, *55*(4), 1772–1782.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88–124.

Lesh, R. A., & Doerr, H. M. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. New York, NY: Routledge.

Lesh, R. A., Post, T., Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Levin, D. M., Hammer, D., & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education*, *60*(2), 142–154.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Liu, P. (2016). Technology integration in elementary classrooms: Teaching practices of student teachers. *Australian Journal of Teacher Education*, *41*(3), 87–104.

Lloyd, G., & Frykholm, J. (2000). How innovative middle school mathematics can change prospective elementary teachers' conceptions. *Education*, *120*, 575-580.

Lui, A. M., & Bonner, S. M. (2016). Preservice and inservice teachers' knowledge, beliefs, and instructional planning in primary school mathematics. *Teaching and Teacher Education*, *56*, 1-13. doi.org/10.1016/j.tate.2016.01.015

Mc Alister, M., Dunn, J., & Quinn, L. (2005). Student teachers’ attitudes to and use of computers to teach mathematics in the primary classroom. *Technology, Pedagogy, and Education*, *14*(1), 77–105. doi.org/10.1080/14759390500200194

Medrano, J. (2012). *The effect of cognitively guided instruction on primary students’ math achievement, problem-solving abilities, and teacher questioning*. (Doctoral dissertation). Retrieved from Proquest Dissertation and Thesis. Arizona State University.

Merriam, S. B., (2009). *Qualitative research: A guide to design and implementation.* San Francisco, CA; Jossey-Bass.

Miles, M. B., Huberman, A. M., Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook.* Thousand Oaks, CA: Sage.

Mizala, A., Martinez, F., Martinez, S. (2015). Pre-service elementary school teachers' expectations about student performance: How their beliefs are affected by their mathematics anxiety and student's gender. *Teacher and Teacher Education, 50,* 70-78.

Molebash, P. (2004). Preservice teacher perceptions of a technology-enriched methods course. *Contemporary Issues in Technology and Teacher Education, 3*(4), 412-432.

Morris, A. K., & Hiebert, J. (2017). Effects of teacher preparation courses: Do graduates use what they learned to plan mathematics lessons*? American Educational Research Journal*, *54*(3), 524-567.

Moscardini, L. (2014). Developing equitable elementary mathematics classrooms through teachers learning about children's mathematical thinking: Cognitively Guided Instruction as an inclusive pedagogy. *Teaching and Teacher Education*, *43*, 69–79.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematic*s. Reston, VA: NCTM. Retrieved from: https://www.nctm.org/uploadedFiles/Standards and Positions/PSSM ExecutiveSummary.pdf NRC 2001

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

National Governors Association Center for Best Practices, & Council of Chief State School Officers (2010). *Common core state standards for mathematics*. Washington, DC.

Newton, K. J., Leonard, J., Evans, B. R., & Eastburn, J. A. (2007). Preservice elementary teachers' mathematics content knowledge and teacher efficacy. *School Science and Mathematics*, *112*(5), 289–299. doi.org/10.1111/j.1949-8594.2012.00145.x

Nyaumwe, L. (2004). The impact of full time student teaching on preservice teachers' conceptions of mathematics teaching and learning. *Mathematics Teacher Education and Development*, *6*, 19–30.

Oonk, W., Verloop, N., & Gravemeijer, K. P. E. (2015). Enriching practical knowledge: exploring student teachers' competence in integrating theory and practice of mathematics teaching. *Journal for Research in Mathematics Education*, *46*(5), 559–598.

O'Pry, L. K., & Paese, P. C. (1987). *A cooperating teacher's intervention strategy on a student teacher.* Southwest Texas State University. Retrieved from: https://eric.ed.gov/?id=ED317492

Osana, H. P., Lacroix, G. L., Tucker, B. J., & Desrosiers, C. (2006). The role of content knowledge and problem features on preservice teachers' appraisal of elementary mathematics tasks. *Journal of Mathematics Teacher Education*, *9*(4), 347–380.

Osana, H. P., & Royea, D. A. (2011). Obstacles and challenges in preservice teachers' explorations with fractions: A view from a small-scale intervention study. *The Journal of Mathematical Behavior*, *30*(4), 333-352.

Ozogul, G., Olina, Z., & Sullivan, H. (2008). Teacher, self and peer evaluation of lesson plans written by preservice teachers. *Educational Technology Research and Development*, *56*(2), 181-201.

Paese, P. C. (1984). The effects of cooperating teacher intervention and a self-assessment technique on the verbal interactions of elementary student teachers. *Journal of Teaching in Physical Education*, *3*(3), 51–58.

Patton, M. Q. (2015). *Qualitative research and evaluation methods* (2nd ed.) Los Angeles, CA: Sage.

Petancio, J. A. M., & Bonotan, A. M. (2018). The realities of teaching elementary mathematics by student teachers: A phenomenological probe. *Asia Pacific Journal of Multidisciplinary Research*, *6*(1), 59–65.

Peterson, B. E., Leatham, K. R. (2009). Learning to use students' mathematical thinking to orchestrate a class discussion. In L. Knott (Ed.), *The role of mathematics discourse in producing leaders of discourse* (pp. 99–128). Charlotte, NC: Information Age Publishing.

Philipp, R. (2000). Questions to ask and issues to consider while supervising elementary mathematics student teachers. *Teacher Education Quarterly*, *27*(1), 69–87.

Philipp, R. A., Ambrose, R., Lamb, L. L. C., Sowder, J. T., Schappelle, B. P., Sowder, L., (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education*, *38*(5), 438-476.

Porter, A. (2002). Measuring the content of instruction: Uses in research and practice. *Educational Researcher, 31*(7), 3-14.

Raizen, S. A., & Michelsohn, A. M. (1994). The future of science in elementary schools. *Educating prospective teachers*. San Francisco, CA: Jossey-Bass.

Rayner, V. (2015). *Developing preservice teachers' professional noticing of students' learning.* (Doctoral dissertation). Retrieved from: https://link-springer-com.ezp2.lib.umn.edu/journal/10212/28/4

Roche, A. (2013). Choosing, creating, and using story problems: some helpful hints. *Australian Journal of Primary Mathematics, 18(*1), 30-35.

Rose, J., & Rogers, S. (2012). Principles under pressure: Student teachers' perspectives on final teaching practice in early childhood classrooms. *International Journal of Early Years Education*, *20*(1), 43-58. doi:10.1080/09669760.2012.664472

Rosenkränzer, F., Kramer, T., Hörsch, C., Schuler, S., & Rieb W. (2016). Promoting student teachers' content related knowledge in teaching systems thinking: Measuring effects of an intervention through evaluating a videotaped lesson. *Higher Education Studies*, *6*(4), 156. doi.org/10.5539/hes.v6n4p156

Roth McDuffie, A. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education*, *7*(1), 33–61.

Rusznyak, L., & Walton, E. (2011). Lesson planning guidelines for student teachers: A scaffold for the development of pedagogical content knowledge. *Education as Change*, *15*(2), 271-285. doi:10.1080/16823206.2011.619141

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education*, *16*(5), 379–397.

Scheeler, M. C., McAfee, J. K., Ruhl, K. L., & Lee, D. L. (2006). Effects of corrective feedback delivered via wireless technology on preservice teacher performance and student behavior. *Teacher Education and Special Education*, *29*(1), 12–25. doi.org/10.1177/088840640602900103

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grows (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334- 370). New York, NY: Macmillan.

Schoenfeld, A. H., Sloane, A. H. (2016). *Mathematical thinking and problem solving.* Philadelphia, PA: Routledge.

Schwartz, C., Walkowiak, T. A., Poling, L., Richardson, K., & Polly, D. (2018). The nature of feedback given to elementary student teachers from university supervisors after observations of mathematics lessons. *Mathematics Teacher Education and Development*, *20*(1), 62–85.

Seethaler, P. M., Fuchs, L. S., Fuchs, D., Compton, D. L. (2011). Predicting first graders' development of calculation vs. word-problem performance: The role of dynamic assessment. *Journal of Educational Psychology*, *104*(1), 224-234.

Shambaugh, N., & Magliaro, S. G. (2006). *Instructional design: A systematic approach for reflective practice.* Boston, MA: Pearson.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational* Review, 57, 1-22.

Siebert, C. J., Clark, A., Kilbridge, A., & Peterson, H. (2006). When preservice teachers struggle or fail: Mentor teachers' perspectives. *Education 3,* 409-422.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*(6), 499–511. doi.org/10.1007/s10857-008-9089-5

Sleep, L., & Boerst, T. A. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education*. *28*(7), 1038–1048.

Smith, L. K., & Gess Newsome, J. (2004). Elementary science methods courses and the national science education standards: Are we adequately preparing teachers? *Journal of Science Teacher Education. 15*(2), 91-110.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions.* Reston, VA: Reston.

Soto-Johnson, H., Liams, M., Oberg, T., Boschmans, B., & Hoffmeister, A. (2010). Promoting preservice elementary teachers' awareness of learning and teaching mathematics conceptually through "KTEM". *School Science & Mathematics*, *108*(8), 345–354. doi-org.ezp1.lib.umn.edu/10.1111/j.1949-8594.2008.tb17849.x

Spitzer, S. M., Phelps, C. M., Beyers, J. E. R., Johnson, D. Y., & Sieminski, E. M. (2011). Developing prospective elementary teachers' abilities to identify evidence of student mathematical achievement. *Journal of Mathematics Teacher Education*, *14*(1), 67–87.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340.

Stengel, B. S., & Tom, A. R. (1996). Changes and choices in teaching methods. *The teacher educator's handbook: Building a knowledge base for the preparation of teachers*, 593-619. San Francisco, CA: Jossey-Bass

Stock, P., Desoete, A., & Roeyers, H. (2009). Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities? *Learning and Individual Differences*, *19*(4), 419–422.

Stockero, S., Peterson, B. E., Leatham, K. R., & Van Zoest, L. R. (2014). The “most” productive student mathematical thinking. *The Mathematics Teacher, 108*(4), 308-312.

Stockero, S. L., Rupnow, R. L., & Pascoe, A. E. (2015). Noticing student mathematical thinking in the complexity of classroom instruction. In T. G. Bartell, K. N. Bieda, R. T. Putnam, K. Bradford, & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 820–827). East Lansing, MI: Michigan State University.

Stripling, C., Ricketts, J. C., Roberts, T. G., & Harlin, J. F. (2008). Preservice agricultural education teachers’ sense of teaching self-efficacy. *Journal of Agricultural Education*, *49*(4), 120-130. doi:10.5032/jae.2008.04120

Sun, J., & van Es, E. A. (2015). An exploratory study of the influence that analyzing teaching has on preservice teachers’ classroom practice. *Journal of Teacher Education*, *66*(3), 201–214. doi.org/10.1177/0022487115574103

Swars, M. E., Smith, S. L., Smith, S. Z., Hart, L. C., Haardorfer, R. (2012). Effects of an additional mathematics content course on elementary teachers’ mathematical beliefs and knowledge for teaching. *Action in Teacher Education*, *34*(4), 336-348.

Swars, S. L., Smith, S. Z., Smith, M. E., & Hart, L. C. (2006). Elementary preservice teachers’ changing pedagogical and efficacy beliefs during a developmental teacher preparation program. *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Vol. 2-548.

Taplin, M. (1998). Preservice teachers' problem solving processes. *Mathematics Education Research Journal, 10*(3), 59-76.

Taylan, R. D. (2016). The relationship between pre-service mathematics teachers' focus on student thinking in lesson analysis and lesson planning tasks. *International Journal of Science and Mathematics Education*, *16*(2), 337–356.

Temiz, T. & Topcu, M. S., (2013). Preservice teachers' efficacy beliefs and constructivist-based teaching practice. *European Journal of Psychology of Education, 28*(4), 1435-1452.

Thompson, P. W., Carlson, M., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education*, *10*(4-6), 415–432. doi:10.1007/s10857-007-9054-8.

Ulu, M. (2016). A structural equation model to explain the effect of fluent reading, literal comprehension and inferential comprehension levels of elementary school 4th grade students on success in problem solving. *Education and Science, 41*(186), 93-117. doi:10.15390/EB.2016.6303

Upadhyay, B. (2008). Elementary students' retention of environmental science knowledge: Connected science instruction versus direct instruction. *Journal of Elementary Science Education, 20*(2), 23-37.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics* Education, 30, 89–110.

Valencia, S. W., Martin, S. D., Place, N. A. & Grossman, P. (2009). Complex interactions in student teaching: Lost opportunities for learning. *Journal of Teacher Education, 60*(3), 304-322.

van Ingen, S., Alvarez McHatton, P., & Vomvoridi-Ivanovic, E. (2016). How do preservice teachers understand the use of research to inform practice: Foundational knowledge for bridging the gap. *Action in Teacher Education*, *38*(2), 175-189. doi:10.1080/01626620.2016.1155094

Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of instances of student mathematical thinking that are worth building on in whole-class discussion. *Mathematical Thinking and Learning*, *19*(1), 33–54. doi.org/10.1080/10986065.2017.1259786

Webb, N. M., Franke, M. L., De, T., Chan, A. G., Freund, D., Shein, P., Melkonian, D. K. (2009). Explain to your partner: Teachers' instructional practices and students' dialogue in small groups. *Cambridge Journal of Education, 39*(1), 49-70.

West Burns, R., Jacobs, J., & Yendol-Hoppey, D. (2016). The changing nature of the role of the university supervisor and function of preservice teacher supervision in an era of clinically-rich practice. *Action in Teacher Education, 38*(4), 410-425. doi:10.1080/01626620.2016.1226203

Wiggins, R. A., & Clift, R. T., (1995). Oppositional pairs: Unresolved conflicts in student teaching. *Action in Teacher Education*, *17*(1), 9-19. doi:10.1080/01626620.1995.10463226

Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2002). Teacher preparation research: An insider's view from the outside. *Journal of Teacher Education*, *53*(3), 190–204. doi.org/10.1177/0022487102053003002

Yin, R. K. (2012). *Applications of case study research*. Los Angeles, CA: Sage.

Zanting, A., Verloop, N., Vermunt, J. (2001). Student teachers' beliefs about mentoring and learning to teach during teaching practice. *British Journal of Educational Psychology, 71*(1), 57-80.

Zeichner, K. (2007). Professional development schools in a culture of evidence and accountability. *School-University Partnerships*, *1*(1), 9-17.

Zembal-Saul, C., Krajcik, J., & Blumenfeld, P. (2002). Elementary student teachers' science content representations. *Journal of Research in Science Teaching, 39*(6), 443–463. doi.org/10.1002/tea.10032

Appendices

The appendices include all material that was used in the study. This includes the self-efficacy pre-assessments; college lesson plan boilerplate; CGI elements, problem types, and solution strategies; interview protocol; and omitted research questions.

Appendix A: Participant Self-Administered Pre-Assessment
Mathematics Self-Efficacy

Name _______________

How do you feel about mathematics?

What is your math ACT score?

When did you last have a math class?

What experiences have you had learning mathematics?

Was there a life moment that got you excited about math (i.e. learned how to relate math to everyday things)?

Describe a favorite math teacher:

What characteristics does a good math teacher have?

What are your feelings about teaching elementary mathematics?

What are your feelings about teaching math in your student teaching placement?

Should elementary mathematics be taught a certain way or with certain curricula?

Appendix B: Participant Self-Administered Pre-Assessment
Knowledge of CGI

Name ______________

1. CGI helps students learn through
a. Memorizing procedures
b. Exploring situations
c. Memorizing problems
d. Practicing procedures

2. Using CGI often involves
a. Acting out a story
b. Memorizing answers
c. Reflecting on ways to solve a problem
d. Rewriting a problem to make it a story

3. CGI uses problem types to
a. Help students memorize how to solve problems
b. Clarify characteristics of problems
c. Make practicing problems more theoretical
d. Start students' math imagination

4. CGI problem types include
a. Join problems
b. Mixed type problems
c. Algorithm problems
d. Multi-step problems

5. Children's solution strategies
a. Show students' intuitive abilities to solve problems
b. Are often contradictory and usually incorrect
c. Should be narrowed to one strategy
d. Show students' inaccurate language in describing the solution

6. CGI teachers often
a. Ask students to remember the steps to solve a problem
b. Ask students if they can solve a problem in several ways
c. Ask students about key words in the problem as ways to help solve it
d. Ask students if the problem should be solved in a certain way

7. CGI teachers often
a. Expect students to focus on the answer of a problem
b. Expect students to solve problems incorrectly
c. Expect students to be confused about word problems
d. Expect a range of solution strategies

8. CGI teachers
a. Value efficiency as students solve problems
b. Value student thinking as ways to solve problems
c. Value their knowledge of a problem over students' thinking
d. Value understanding of procedures when solving problems

9. CGI teachers listen to students' thinking
a. To gather information about the math problem
b. To encourage students' use of memorized strategies
c. To steer students to efficient algorithms
d. To guide students in solving a math problem

10. CGI teaches that children's solution strategies to appropriate math problems
a. Are often based on misconceptions
b. Are often based on concepts not associated with the problem
c. Are often based on concepts directly related to the problem
d. Are often based on clues given by the teacher or other student

11. CGI teachers value sense making in mathematics
a. Because sense making is the goal of doing mathematical work
b. Because students want to get their problems correct and completed
c. Because children's mistakes are often not connected to making sense of the problem
d. Because leveraging students' reasoning often leads to incorrect solution strategies

12. CGI solution strategies
a. Range from direct modeling to flexible choice of strategies
b. Are different from strategies children use
c. Are equally sophisticated
d. Follow specific patterns when solving problems

13. Students' use of finger counting
a. Are the same for every stage of math development
b. Varies from child to child
c. Is not usually used by young children learning to count
d. Indicates a student's lack of number sense

Appendix C: College Boilerplate Lesson Plan

Lesson Title: Teacher: Grade/Subject: First grade mathematics Number of Students:	
Standards Addressed	What P-12 state standards are addressed? 1.1.2.1 Use words, pictures, objects, length-based models (connecting cubes), numerals, and number lines to model and solve addition and subtraction problems in part-part-total, adding to, taking away from, and comparing situations.
Learning Objectives	What will students be able to do after the lesson? ● Moving from the Benchmarks to students can – "I can" – statements. ● What will students know and be able to do? -------------------- I can
Diagnostic Assessment	What do students already know about the topic? How will you build on their prior knowledge and experiences? How will you determine what they know? ------------------- Students have experience with
Individual Differences	List one or two differences that exist in your class or hypothetical students (if none exist): How will you accommodate individual differences in your class for the following students (if present): ELL, Special Education, Cultural/Ethnic groups, Gifted & Talented, reading disabilities, etc.? How will you differentiate your instruction for varied learners? -------------------
Materials, Technology, and/or Special Arrangements	List appropriate materials and technology used to enhance student engagement and learning. List any special arrangements needed. -----------------------

Behavior Expectations	Describe expectations for student interactions, classroom procedures, and individual accountability. --------------------------- Students will be

Personal Goals	Identify personal goals related to Standards of Effective Practice (SEP) specific to the learning targets and learners in this lesson. (These are your personal goals, not student goals.) ----------------------------
Anticipatory Set	Describe how you will engage students through introductory activity (Introduce learning targets, ask essential questions, connect with student experience, engage with inquiry) ----------------------- Students will
Academic Language	What specific academic language will be used in this lesson? What connotations and denotations do students need to know for this lesson? ----------------------
Activities: Guided Practice Key questions Modeling Scripting	What sequence of teaching and learning experiences will equip students to engage with, develop, and demonstrate the desired objectives? (List the activities, guided practice, key questions, and demonstrations/modeling to help students construct meaning. Include introductory scripts or prompts. Detail sequence; note timing and transitions.) Develop appropriate reinforcements of learning targets (seatwork, projects, homework). ================== Students will Students will

Formative Assessment	Use varied and appropriate formal and informal assessments to check for understanding and measure student outcomes throughout the lesson. -----------------------
Summative Assessment	List all summative assessments included in this lesson. How will you know if the learning targets/objectives have been met? How does this lesson connect with the larger unit of study? Include a teacher created rubric (if applicable) Include a student created rubric (if applicable) -----------------------
Closure	Provide an activity that reinforces learning, makes connections, and/or assesses learning related to essential objectives. Closing a lesson is an activity in itself, not just a summary. ------------------------

Appendix D: CGI Elements - Key Elements of Cognitively Guided Instruction

CGI Element	Description
Expects strategies	PSET expects students to pursue solution strategies
Invites sharing	PSET invites students to share solution strategies
Problem types	PSET uses CGI problem types
Word problems	PSET uses word problems to build number sense
Broadcasting	PSET broadcasts student thinking to the class
Presents problems	Presents problems without modeling any solution strategies
Invites reflections	Invites students to reflect on other students' thinking
Invites multiple	Invites multiple strategies from students
PSETs learn	PSETs learn from listening to student thinking
PSETs struggle	PSETs willing to struggle to understand students' strategies
Intentional listening	Teachers intentionally listen to student thinking
Starting points	Builds on student starting points
Intuition	PSET uses students' intuitive problem-solving abilities
Flexible range	PSETs use flexible range of numbers when assigning problems
Completion	Pursue students' thinking to completion

CGI Element	Description
Respect	Teachers communicate respect for others' thinking
Correlate	PSETs correlate problem types with solution strategies
Invented	Invented algorithms

Appendix E: CGI Problem Types

<table>
<tr><td colspan="2">Result Unknown

Megan had 3 markers. Robert gave her 2 more markers. How many markers does Megan have altogether?</td><td colspan="2">Change Unknown

A CD rack holds 8 CDs. There are 5 CDs already in the rack. How many more CDs can be put on the rack?</td><td colspan="2">Start Unknown

Carla has some marbles. She bought 3 marbles. Now she has 7 marbles. How many did she start with?</td></tr>
<tr><td colspan="2">Result Unknown

Paco had 9 cookies. He ate 6 of them. How many cookies did Paco have left?</td><td colspan="2">Change Unknown

Kelly has 6 Nintendo games. How many does she need to give away so that she will have 3 games left?</td><td colspan="2">Start Unknown

Connie had some marbles. She gave 2 to Juan. Now she has 5 marbles left. How many did she have to start with?</td></tr>
<tr><td colspan="3">Whole Unknown

Connie has 5 red markers and 8 blue markers. How many markers does she have altogether?</td><td colspan="3">Part Unknown

Iesha has 14 books. 6 are about school and the rest are about sports. How many books about sports does Iesha have?</td></tr>
<tr><td colspan="2">Difference Unknown

James has 12 balloons. Amy has 7 balloons. How many more balloons does James have than Amy?</td><td colspan="2">Compare Quantity Unknown

Connie has 13 marbles. Juan has 5 more marbles than Connie. How many marbles does Juan have?</td><td colspan="2">Referent Set Unknown

Sean has 13 whistles. He has 5 more whistles than Charles. How many whistles does Charles have?</td></tr>
<tr><td colspan="2">Multiplication

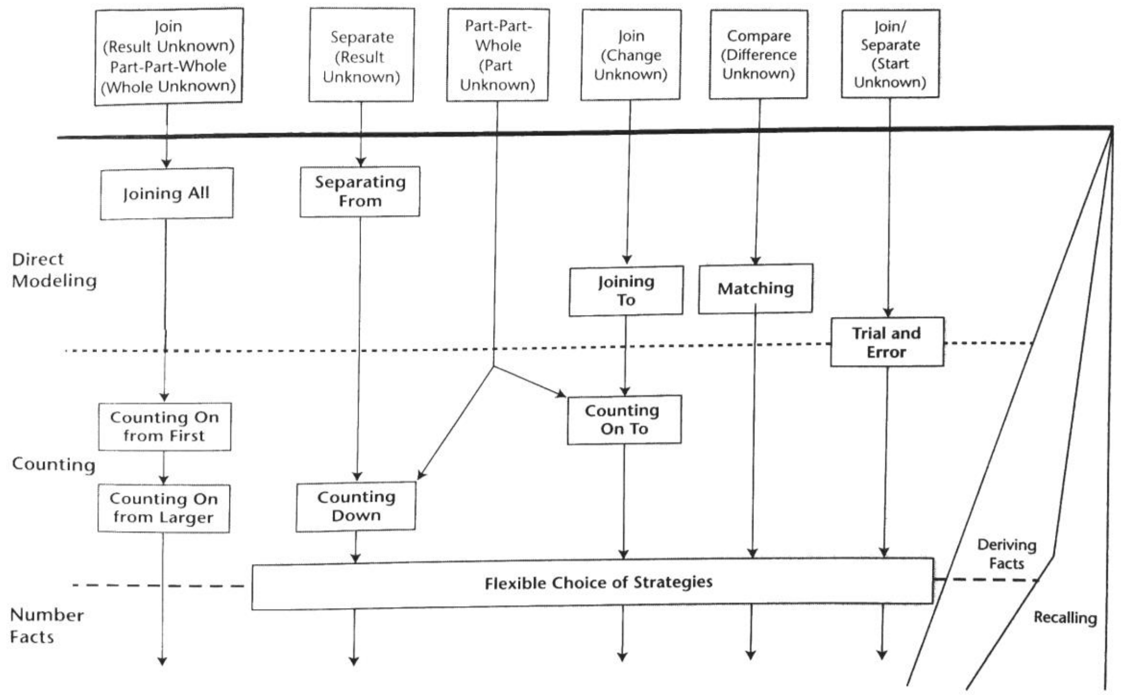
Robin has 3 packages of gum. There are 4 pieces in each package. How many pieces of gum does Robin have?</td><td colspan="2">Measurement Division

I have 15 cents to buy candy. If each gumdrop costs 3 cents, how many gumdrops can I buy?</td><td colspan="2">Partitive Division

21 people are going to the zoo. There are 3 cars to take people to the zoo. How many will go in each car if the same number go in each car?</td></tr>
</table>

CGI problem types with examples. These type names are referenced in this research, although these same problem types are called by other names in many mathematical materials. Adapted with permission from *Children's Mathematics*, Second Edition: *Cognitively Guided Instruction* by Carpenter, Fennema, Franke, Levi, and Empson. Copyright © 2015 by Carpenter et al. Published by Heinemann, Portsmouth, NH.

Appendix F: CGI Solution Strategies



CGI solution strategies chart showing the three levels of strategies children use to solve word problems: direct modeling (the least sophisticated), counting, and number facts/derived facts (the most sophisticated). Reprinted with permission from *Children's Mathematics*, Second Edition: *Cognitively Guided Instruction* by Carpenter, Fennema, Franke, Levi, and Empson. Copyright © 2015 by Carpenter et al. Published by Heinemann, Portsmouth, NH.

Appendix G: Interview Protocol

This semi-structured interview will form a baseline of PSET's math instruction. It will help establish the mathematical thinking and pedagogical practices of PSET before the intervention. It will reveal the thinking of each PSET and will be compared to the next two phases of the study.

How did the lesson go?

What was your math focus in this lesson?

What math ideas did children talk about?

What CGI ideas, if any, emerged in this lesson?

What CGI strategies, if any, did you observe the students using?

What did you observe about children trying to make sense of their work?

Why did you follow up on students' thinking?

Why did you follow up on students' problem solving strategies?

What did children say about other children's math talk?

What math sense making emerged with students?

Appendix H: Omitted Research Questions

4. What counting strategies or problem solving strategies emerge from students in lesson constructed before, during, and after an LPI lesson in early number?
5. What instances emerge, if any, of students extending counting strategies to solve problems in lessons constructed before, during, and after an LPI lesson in early number?
6. What instances emerge, if any, of students commenting on or using other students' strategies in lessons constructed before, during, and after an LPI lesson in early number?